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Abstract:

Policy makers in the EU and elsewhere are concerned that unilateral carbon pricing induces carbon leakage through relocation of emission-intensive and trade-exposed industries to other regions. A common measure to mitigate such leakage is to combine an emission trading system (ETS) with output-based allocation (OBA) of allowances to exposed industries. We first show analytically that in a situation with an ETS combined with OBA, it is optimal to impose a consumption tax on the goods that are entitled to OBA, where the tax is equivalent in value to the OBA-rate. Then, using a multi-region, multi-sector computable general equilibrium (CGE) model calibrated to empirical data, we quantify the welfare gains for the EU to impose such a consumption tax on top of its existing ETS with OBA. We run Monte Carlo simulations to account for uncertain leakage exposure of goods entitled to OBA. The consumption tax increases welfare whether the goods are highly exposed to leakage or not. Thus, policy makers in regions with OBA can only gain by introducing the consumption tax. It can hence be regarded as smart hedging against carbon leakage.

Keywords: Carbon leakage; output-based allocation; consumption tax

JEL classification: D61, F18, H23, Q54

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1. Introduction

Although the Paris Agreement entails that all signatory countries should mitigate greenhouse gas emissions, the stringency of climate policies varies substantially across countries. The European Union has been a frontrunner in greenhouse gas emissions pricing, initiating its EU Emission Trading Scheme (ETS) in 2005. The EU ETS regulates about half of the greenhouse gas emissions in the EU, mainly emissions from large energy-intensive installations in the electricity and manufacturing sectors. From the very start of the EU ETS, policy makers in the EU have been concerned about carbon leakage associated with the relocation of emission-intensive and trade-exposed (EITE) production to countries with less stringent climate policies. Hence, large amounts of free allowances have been granted to EITE industries, with carbon leakage as the explicit motive. Allocation of allowances is proportional to the individual installation's output, so-called output-based allocation (OBA). Similar allocation schemes are also applied in other ETS (Meunier et al., 2017).

There is a large literature showing that implementing OBA tends to reduce leakage and improve competitiveness compared to carbon pricing alone (e.g., Zhang, 2012). However, this comes with a negative side effect, as OBA simultaneously leads to excessive use of EITE goods. The explanation is that OBA works as an implicit subsidy to EITE production, which is particularly distortive for sectors that are little exposed to leakage after all. Hence, border carbon adjustments (BCA), in particular carbon tariffs on imports of EITE goods, have been regarded in the literature as a more cost-effective instrument to mitigate global emissions (Böhringer et al., 2014). Whereas OBA stimulates domestic production, carbon tariffs constrain foreign supply. BCA are more contentious though in terms of WTO compatibility, and while figuring prominently in anti-leakage climate policy proposals of several regions have so far not been implemented.[1]

More recently, some studies suggest to tax domestic use of EITE goods as a supplement to OBA. In particular, Böhringer et al. (2017) analyze the effects of

[1] For example, border measures were included in the American Clean Energy and Security Act of 2009 that passed the U.S. Congress but not the Senate (Fischer and Fox, 2011). Border measures were also put forward by the EU Commission (2009) as a possible future alternative to free allowance allocation.

imposing a so-called consumption tax on all use (not only final consumption) of EITE goods in a situation where carbon pricing and OBA have already been implemented. They show, both analytically and within a stylized numerical model, that such a tax is likely to be welfare enhancing. The intuitive reasoning behind is that the consumption tax alleviates the negative effects of OBA, that is, the excessive use of EITE goods. Moreover, they also show that under certain conditions such a policy combination will in fact be equivalent with carbon pricing combined with full BCA (thus avoiding potential WTO disputes of anti-leakage climate policy measures).

The negative effects of OBA are particularly large if the leakage exposure is limited (Böhringer et al., 2017). For policy makers (and others), the actual leakage exposure of industries may be difficult to assess, and trade-exposed industries have incentives to exaggerate the exposure in order to maximize the number of free allowances. Hence, the extent of free allocation may become higher than optimal, which has been shown to be the case in the EU ETS. Martin et al. (2014) conclude that the current allocation in the EU ETS results in "substantial overcompensation for given carbon leakage risk". Whereas a majority of industry sectors receives a high share of free allowances, Sato et al. (2015) find that "vulnerable sectors account for small shares of emission".

In this paper we show – first analytically and then numerically – that supplementing OBA with a consumption tax constitutes smart hedging against carbon leakage. The analytical section concludes that, under certain conditions, it is optimal from a regional and global welfare perspective to implement a consumption tax that is equivalent in value to the OBA-rate.

For our numerical analysis, we use a multi-sector multi-region computable general equilibrium (CGE) model of the global economy. In international trade, goods are distinguished by country of origin following the standard Armington assumption (Armington, 1969): Imported and domestically produced goods of the same kind are treated as incomplete substitutes. The value of the (Armington) substitution elasticities determines how close substitutes goods produced in different regions are, and hence to what degree the domestic industry is exposed to competition from abroad and to carbon leakage. Thus, varying these elasticities means varying the leakage exposure in the model. To reflect the uncertainty on empirical estimates for Armington elasticities, we use a Monte Carlo approach based on a probability distribution for the Armington elasticities.

Our simulations for EU climate policy design suggest that imposing a consumption tax as a supplement to OBA is unambiguously positive for the EU. The extent of the welfare gains is negatively correlated with the Armington elasticities: If leakage exposure is lower than assumed, the welfare gains are quite substantial, whereas if leakage exposure is as high as assumed by many policy makers (or even higher), the consumption tax is less advantageous but does no harm either.

The literature on carbon leakage is extensive, going back to seminal theoretical studies by Markusen (1975) and Hoel (1996). Most numerical studies use multi-region and multi-sector CGE models of the global economy (as we do), see e.g. Zhang (2012) for a review. Of particular interest for our analysis of anti-leakage climate policy design are the relatively few studies that examine supplemental consumption taxes. In particular, our paper builds on Böhringer et al. (2017). Compared to that paper, our contribution is twofold. First, whereas Böhringer et al. (2017) show analytically that it is welfare improving to marginally increase the consumption from zero, the current paper derives the optimal *level* of the consumption tax (under certain conditions). Second, Böhringer et al. (2017) apply a stylized CGE model for two symmetric regions, whereas this paper uses a large-scale CGE model based on empirical data to assess EU climate policy design under uncertainty about leakage exposure. Regarding other related studies, Holland (2012) shows analytically, using a one-good model, that a consumption tax can be a supplement to an emission intensity standard, for much the same reasons as pointed out in our paper. Eichner and Pethig (2015a,b) analyze consumption-based taxes, either as an alternative or as a supplement to production-based (emission) taxes, and conclude similarly. An important feature in their model is that emissions can only be reduced by reducing output. In our model, emissions can also be reduced by reducing the emission intensity, which is particularly important from a leakage and competitiveness perspective. Pauliuk et al. (2016) discuss the possibility of including charges for the consumption of carbon-intensive materials in the EU ETS.

The remainder of this paper is organized as follows. In Section 2, we lay out the theoretical model and analyze the optimal consumption tax in a situation where an ETS combined with OBA is already in place. In Section 3, we present our numerical CGE analysis where we quantify the effects of implementing a consumption tax in the context of the EU ETS. Section 4 concludes.

2. Analytical model

Consider a partial equilibrium model with two regions, $j=\{1,2\}$, and three goods *x*, *y* and *z*. Good *x* is emission-free and tradable, good *y* is emission-intensive and tradable, while good *z* is emission-intensive and non-tradable. We interpret *y* as emission-intensive and trade-exposed (EITE) sectors where output-based allocation is considered (e.g., chemicals, metals, and other mineral production), and *z* as sectors where leakage is of less concern (e.g., electricity production and transport). Consumption of *x* in region *j* is denoted $\bar{x}^j$, and similarly for the other goods.

The representative consumer in region *j* has a constant-elasticity-of-substitution (CES) utility function given by:

$$u^j\left(\bar{x}^j,\bar{y}^j,\bar{z}^j\right)=\left(\alpha^{xj}\left(\bar{x}^j\right)^\rho+\alpha^{yj}\left(\bar{y}^j\right)^\rho+\alpha^{zj}\left(\bar{z}^j\right)^\rho\right)^{\frac{1}{\rho}},\quad j=1,2, \tag{1}$$

in which the positive α's represent initial consumption shares, and the substitution elasticity is $1/(1-\rho)$. Assume that *y* is a composite good, consisting of a domestic good *d* and a foreign good *f*, such that:

$$\bar{y}^j=\left(\beta^j\left(\bar{d}^j\right)^\theta+\left(1-\beta^j\right)\left(\bar{f}^j\right)^\theta\right)^{\frac{1}{\theta}}. \tag{2}$$

This formulation allows to differentiate between foreign and domestically produced EITE goods. The Armington elasticity, given by $\sigma=1/(1-\theta)$, determines how close substitutes *d* and *f* are. The goods become perfect substitutes as $\theta\to 1$ ($\sigma\to\infty$), perfect complements as $\theta\to-\infty$ ($\sigma\to 0$), and Cobb-Douglas as $\theta\to 0$ ($\sigma\to 1$). A high

Armington elasticity (θ close to 1) implies a strong potential for carbon leakage. Conversely, the potential for carbon leakage becomes negligible as $\theta \to -\infty$. We have $\rho, \theta \neq 0$ and $\rho, \theta < 1$.

Production of good *x* in region *j* is $x^j = x^{1j} + x^{2j}$, where x^{ij} denotes goods produced in region *j* and sold in region *i*. We use similar notation for goods *z*, *d* and *f*, but omit the redundant region of origin superscript *j* to reduce notational clutter (except when useful in summation signs). Utility does not depend on the country of origin for the emission-free and tradable good *x*. The market equilibrium conditions are:

$$\begin{aligned} & x^1 + x^2 = \overline{x}^1 + \overline{x}^2 \\ & z^j = \overline{z}^j \\ & d = \overline{d}^1 + \overline{d}^2 \\ & f = \overline{f}^1 + \overline{f}^2 \end{aligned} \tag{3}$$

with $j = \{1, 2\}$.

For our analysis, we assume that region 1 undertakes unilateral emission regulation and disposes of three policy instruments: an emission trading regime with permit price t^1, an output subsidy s^1 to production of the domestically produced EITE good *d*, and a domestic consumption tax v^1 on buying EITE goods *d* and *f*. Output-based allocation (OBA) functions similar to an output subsidy, where the implicit subsidy is linked to the price of emission permits. In particular, if the permit sale revenues from EITE producers are fully redistributed back to the EITE producers (not at the firm level but at the aggregate EITE level), the implicit subsidy of OBA is $s^1 = t^1 e^d / d$, a case we will refer to as 100% OBA. The main analysis focuses on the case with no climate policy in region 2, i.e., $t^2 = s^2 = v^2 = 0$, but we consider global emission trading ($t^1 = t^2 > 0$) for comparative statics.

In order to avoid valuing the damages from climate change, we impose that the global emissions are constant across alternative climate policy scenarios. Hence, we require the abating region to adjust its unilateral emissions reduction effort such that a given global emission cap $\overline{E}$ is maintained. If leakage varies across different policy regimes, the effective unilateral emission reduction requirement will be adjusted such that global emissions equal the target $\overline{E}$. Thus, the emission constraint is:

$$\overline{E} = \sum_{j=1,2} \sum_{g \in G} e^{gj}, \tag{4}$$

where e^{gj} denotes emissions from production of good $g \in G = \{x, z, d, f\}$ in region *j*.

Production cost is given by:

$$c^{gj}\left(g^j, e^{gj}\right) = c^g g^j + \frac{\phi^g}{2}\left(\xi^g g^j - e^{gj}\right)^2, \quad j = 1, 2, \tag{5}$$

where c^g, ϕ^g, and ξ^g are constants and $\xi^g g^j$ is business-as-usual (BaU) emissions in the absence of restrictive climate policies. We assume that similar production technologies are available in the two regions, such that the cost functions are identical for the same types of goods (*x*, *y*, and *z*). We have $c^x, c^z > 0$, $c^d = c^f \equiv c^y > 0$, $\xi^x = \phi^x = 0$, $\xi^z, \phi^z > 0$, $\xi^d = \xi^f \equiv \xi^y > 0$ and $\phi^d = \phi^f \equiv \phi^y > 0$. Note that abatement costs are increasing and strictly convex if $\phi^g > 0$.

Let p^{xj}, p^{zj}, p^{dj} and p^{fj} denote the market prices (excluding taxes) of goods *x*, *z*, *d* and *f* in region *j*. We must have $p^{x1} = p^{x2} = c^x \equiv p^x$, because the competitive firms supply the good at a price equal to marginal production cost c^x in both regions.[2] Competitive producers maximize profits:

$$\begin{aligned}
\pi^{xj} &= \arg\max_{x^j}\left(p^x x^j - c^x\left(x^j\right)\right), \quad j = 1, 2, \\
\pi^{zj} &= \arg\max_{z^j, e^{zj}}\left(p^{zj} z^j - c^z\left(z^j, e^{zj}\right) - t^j e^{zj}\right), \quad j = 1, 2, \\
\pi^d &= \arg\max_{\overline{d}^1, \overline{d}^2, e^d}\left(\left(p^{d1} + s^1\right)\overline{d}^1 + \left(p^{d2} + s^1\right)\overline{d}^2 - c^d\left(d, e^d\right) - t^1 e^d\right), \\
\pi^f &= \arg\max_{\overline{f}^1, \overline{f}^2, e^f}\left(p^{f1}\overline{f}^1 + p^{f2}\overline{f}^2 - c^f\left(f, e^f\right) - t^2 e^f\right),
\end{aligned} \tag{6}$$

where we use the market-clearing constraint for the EITE goods in the two last expressions (sales equal consumption). The profits of firms located in region *j* accrue to the representative consumer in that region, and the regulator redistributes the net tax revenue as a lump-sum transfer to the representative consumer. The representative consumer in region *j* solves:

$$\max_{\overline{x}^j, \overline{y}^j, \overline{z}^j} u\left(\overline{x}^j, \overline{y}^j, \overline{z}^j\right), \quad j = 1, 2, \tag{7}$$

[2] It simplifies notation to establish this before formulating the firms' maximization problems in (6); see also Appendix A.

subject to equations (1), (2) and the budget constraint:

$$m^j + A^j \geq p^{xj}\overline{x}^j + p^{zj}\overline{z}^j + \left(p^{dj} + v^j\right)\overline{d}^j + \left(p^{fj} + v^j\right)\overline{f}^j, \quad j = 1, 2, \qquad (8)$$

where m^j denotes an exogenous monetary endowment and A^j is a term that includes firm profits and government income from sale of emission permits and net taxes.[3] We follow the usual assumption that the representative consumers and firms do not consider the redistribution of taxes and profits when choosing consumption and production levels. Utility maximization implies that the budget constraints hold with strict equality (non-satiation in the utility function (1)).

Let $a^{gj} = \xi^g g^j - e^{gj}$ denote the emission reductions for good g in region j caused by lower emission intensity induced by the climate policy regulation. Further, let superscript $* = \{REF, OBA, CTAX, FB\}$ indicate competitive equilibrium values under the regulatory regimes specified in Table 1.[4]

Table 1. Specification of regulatory regimes ($t^j > 0$ indicates emission trading in region j)

	Region 1	Region 2
REF (reference, unilateral emission trading)	$t^1 > 0, s^1 = v^1 = 0$	$t^2 = s^2 = v^2 = 0$
OBA (*REF* with output subsidy)	$t^1 > 0, s^1 > 0, v^1 = 0$	$t^2 = s^2 = v^2 = 0$
CTAX (*OBA* with consumption tax)	$t^1 > 0, s^1 > 0, v^1 > 0$	$t^2 = s^2 = v^2 = 0$
FB ('first-best', global emission trading)	$t^1 = t > 0, s^1 = v^1 = 0$	$t^2 = t > 0, s^2 = v^2 = 0$

[3] We have $A^j = \sum_g t^j e^{gj} + v^j\left(\overline{d}^j + \overline{f}^j\right) - s^j d + \sum_g \pi^{gj}$ (with $s^2 = v^2 = 0$).

[4] Whereas it is reasonable to assume that the global emission cap $\overline{E}$ in equation (4) is equal across the unilateral climate policies (*REF*, *OBA* and *CTAX*), international policies (*FB*) may have more stringent emission caps. Whether or not the global emission cap is more stringent under international agreements does not affect our results, and we keep $\overline{E}$ fixed for simplicity.

Let capital letters indicate consumer prices (including taxes), such that P^x, P^{zj}, P^{dj} and P^{fj} denote the consumer prices of goods *x, z, d* and *f* in region *j* ($P^{x1} = P^{x2} = P^x$). We have the following result, which will be useful in comparing the outcomes of the different regulatory regimes:[5]

Lemma 1 *The interior solution competitive equilibrium is characterized by:*

$$\frac{c^x}{c^z + \xi^z t^{j*}} = \frac{P^{x*}}{P^{zj*}} = \frac{\alpha^{xj}}{\alpha^{zj}} \left(\frac{\bar{z}^{j*}}{\bar{x}^{j*}} \right)^{1-\rho}, \tag{9}$$

$$\frac{c^d + \xi^y t^{1*} - s^1 + v^j}{c^f + \xi^y t^{2*} + v^j} = \frac{P^{dj*}}{P^{fj*}} = \frac{\beta^j}{1-\beta^j} \left(\frac{\bar{f}^{j*}}{\bar{d}^{j*}} \right)^{1-\rho}, \tag{10}$$

$$\frac{c^x}{c^d + \xi^y t^{1*} - s^1 + v^j} = \frac{P^{x*}}{P^{dj*}} = \frac{\alpha^{xj}}{\alpha^{yj}} \frac{1}{\beta^j} \left(\beta^j \left(\bar{d}^{j*} \right)^{\theta} + \left(1-\beta^j\right) \left(\bar{f}^{j*} \right)^{\theta} \right)^{1-\frac{\rho}{\theta}} \frac{\left(\bar{d}^{j*} \right)^{1-\theta}}{\left(\bar{x}^{j*} \right)^{1-\rho}}, \tag{11}$$

$$t^{j*} = \phi^z a^{zj*}, t^{1*} = \phi^y a^{d*}, t^2 = \phi^y a^{f*}, \tag{12}$$

with $j \in \{1,2\}$ *, the global emissions cap (4) and the budget constraints:*

$$\begin{aligned} m^1 + \left(\left(c^d + \xi^y t^{1*} - s^1\right) \bar{d}^{2*} - c^f \bar{f}^{1*} \right) &= c^x \bar{x}^{1*} + c^z \left(z^{1*}, e^{z1*}\right) + c^d \left(d^*, e^{d*}\right) \quad (j=1), \\ m^2 + \left(c^f \bar{f}^{1*} - \left(c^d + \xi^y t^{1*} - s^1\right) \bar{d}^{2*} \right) &= c^x \bar{x}^{2*} + c^z \left(z^{2*}, e^{z2*}\right) + c^f \left(f^*, e^{f*}\right) \quad (j=2), \end{aligned} \tag{13}$$

Proof. See Appendix A.

In equations (9) to (11), the first equalities follow from the producers' first order conditions, whereas the second equalities follow from the consumers' first order conditions. Note that $c^g + \xi^g t^j = c^g + \xi^g \phi^g a^{gj}$ represents the marginal production cost for commodity *g* in region *j* at the emission intensity that follows from the region's emission cap (4). Equation (12) states the familiar result that the emission price t^j equals marginal abatement costs $\phi^g a^{gj}$. Further, the left-hand sides of the budget constraints (13) are monetary endowments m^j plus net income from trade (c^f and $c^d + \xi^y t^1 - s^1$ are the equilibrium prices on imported EITE goods in region 1 and 2, respectively), whereas the right-hand sides are production cost.

[5] The model does not determine production of *x* in each region uniquely, only consumption. This does not matter for the results (there are no emissions or profits from *x*).

We are mostly interested in how the different policy regimes affect production and consumption, i.e., equations (9) to (11). But first it is worth noticing the presence of the subsidy s^1 in the budget constraint (13). Net income from trade for region 1 is reduced by $s^1\overline{d}^{2*}$, i.e., the subsidy times the consumption of *d* in region 2. This may seem surprising, as the subsidy is given to the domestic producer of *d*. The explanation is that the output subsidy creates a wedge between the price on *d* and marginal production cost. For example, in the case of 100% *OBA* ($s^1 = \xi^y t^1$), marginal production cost is $c^d + \xi^y t^1$, whereas the export price is $p^{d2} = c^d$. Hence, the representative consumer in region 1 (which owns the firms and collects net tax revenues in region 1) sells the domestic EITE goods with negative profits.[6] That is, the *OBA* subsidy does not only distort the relative prices, it also involves subsidizing foreign consumption of the domestically produced EITE good *d*. The cost of this subsidy, $s^1\overline{d}^2$, is captured as a monetary transfer from region 1 to region 2 in the budget constraints (13).[7]

When it comes to consumption in the non-regulating region 2, we observe from Lemma 1 that relative prices and hence relative consumption levels are equal under *OBA* and *CTAX*, because the consumption tax v^1 only affects prices in the domestic region 1 ($v^2 = 0$ in equations (10) and (11)).[8]

We will henceforth focus on the special cases of *OBA* and *CTAX* where $s^1 = \zeta^y t^1$ (100% *OBA*) and $v^1 = s^1$. Assume first that there is unilateral emission trading (*REF*) in Region 1. Then we get from equation (10) when comparing with the first-best (*FB*):

$$\frac{P^{dj,REF}}{P^{fj,REF}} = \frac{c^d + \xi^y t^{1,REF}}{c^f} > \frac{c^d + \xi^y t^{FB}}{c^f + \xi^y t^{FB}} = \frac{P^{dj,FB}}{P^{fj,FB}}, \quad j = 1,2, \tag{14}$$

We see that unilateral emission trading causes too large a share of the composite EITE good *y* originating from abroad, relative to the first-best (*FB*) allocation, with associated carbon leakage. This observation is the motivation for implementing

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6 The profit maximizing firm would not sell *d* at price below $c^d + \xi^y t^1$ without the subsidy.

7 Note that $s^1\overline{d}^1$ cancels out in the budget constraint (13) for region 1, because the representative consumer in this case subsidizes domestic consumption $\overline{d}^1$, as opposed to foreign consumption $\overline{d}^2$. Remember that $\xi^y t^1$ in (13) represents abatement costs.

8 This result relies on the constant-returns-to-scale cost function.

output-based allocation, which implies a shift from *REF* regulation to *OBA* regulation as defined in Table 1. The policy shift ameliorates the competitiveness tension highlighted by (14), because we then have $P^{dj,OBA}/P^{fj,OBA} = c^d/c^f$. Inspection of Lemma 1 shows that *OBA* is a two-edged sword, however. That is, whereas *OBA* reduces the carbon leakage highlighted by equation (14), it also induces excessive consumption of the domestically produced EITE good *d* (because of the *OBA* subsidy to production).[9] This follows from equation (11), which gives:

$$\frac{P^{x,OBA}}{P^{dj,OBA}} = \frac{c^x}{c^d} > \frac{c^x}{c^d + \xi^y t^{FB}} = \frac{P^{x,FB}}{P^{dj,FB}}, \quad j = 1,2, \tag{15}$$

We notice that *OBA* implies too much consumption of the emission-intensive *d* good, relative to the clean good *x*, as compared to the first-best case. Moreover, dividing equation (11) by equation (9), we get:

$$\frac{P^{zj,OBA}}{P^{dj,OBA}} = \frac{c^z + \xi^z t^{OBA}}{c^d} > \frac{c^z + \xi^z t^{FB}}{c^d + \xi^y t^{FB}} = \frac{P^{zj,FB}}{P^{dj,FB}}, \quad j = 1,2, \tag{16}$$

which implies too much consumption of *d* relative to the non-tradable emission-intensive good *z*, as compared to the first-best case. Hence, moving from *REF* to *OBA* may increase or decrease the utility of region 1, depending on the functional forms.

Interestingly, the too high price ratios $P^{x,OBA}/P^{dj,OBA}$ and $P^{zj,OBA}/P^{dj,OBA}$ under *OBA* (cf. (15) and (16)) are counteracted in the domestic market by introducing a consumption tax v^1 on domestic consumption of the EITE goods. Furthermore, this consumption tax does not increase carbon leakage through the competitiveness channel, because the consumption tax is levied on both domestic and foreign EITE goods (cf. (10)). Finally, the consumption tax ameliorates the negative externality caused by unregulated emissions from foreign production of domestically consumed EITE goods $\overline{f}^1$. Indeed, assume, for the sake of our argument, that the permit price under *CTAX* is equal to the permit price under global emission trading ($t^{1,CTAX} = t^{FB} = t$).[10] Then it can

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[9] It is well-known that OBA distorts relative prices and may cause excessive production of the EITE goods; see, e.g., Böhringer and Lange (2005).

[10] Equation (12), the emissions cap (4) and strictly convex abatement costs (5) imply that the equilibrium price on emission permits is higher under *REF*, *OBA* or *CTAX* than under *FB* if $\overline{E}$ are equal across the regulatory regimes. Hence, equal permit prices in *CTAX* and *FB* imply lower global emissions in the latter regime.

be shown that a *CTAX* regime with $v^1 = s^1$ replicates the relative prices in the home region under global emission trading, i.e., we have from Lemma 1:

$$\frac{P^{x,CTAX}}{P^{z1,CTAX}} = \frac{c^x}{c^z + \xi^z t} = \frac{P^{x,FB}}{P^{z1,FB}}, \frac{P^{d1,CTAX}}{P^{f1,CTAX}} = \frac{c^d + \xi^y t}{c^f + +\xi^y t} = \frac{P^{d1,FB}}{P^{f1,FB}}, \frac{P^{x,CTAX}}{P^{d1,CTAX}} = \frac{c^x}{c^d + \xi^y t} = \frac{P^{x,FB}}{P^{d1,FB}}, \tag{17}$$

Note, however, that *production* of the domestic EITE good *d* under *CTAX* is still too high, because the domestic consumption tax is not applied to exports of *d*. It follows that *CTAX* approximates the global emission trading allocation for *consumption* in the home region if the emission price in the home region is the same in the two regimes.[11]

We have the following result:

Proposition 1. *Consider the competitive equilibrium characterized by Lemma 1 with unilateral emission trading and 100% OBA; i.e., we have* $s^1 = \xi^y t^1 > 0$ *and* $t^2 = s^2 = v^2 = 0$. *Assume that a consumption tax* $v^1 \geq 0$ *is feasible. Then, setting* $v^1 = s^1$ *maximizes welfare in region 1 (given no other changes to the regulatory regimes in regions 1 and 2).*

Proof. See Appendix A

Proposition 1 implies that welfare in Region 1 can be increased by coupling an existing *OBA* regime with a consumption tax equal to the implicit *OBA* subsidy.[12] In fact, the optimal level of the consumption tax is identical to the *OBA* subsidy, given the model assumptions outlined above. Note that terms-of-trade effects do not appear in the analytical model, given 100% *OBA* at home and no climate policy abroad. The reason is that constant returns to scale makes export and import prices exogenous. In our CGE analysis below, we will see that terms-of-trade effects may be quite important when considering regional welfare.

Proposition 1 has the following corollary:

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[11] This implies that output-based rebating (where the emission price is fixed but global emissions are endogenous) coupled with a consumption tax can replicate the relative prices under a global emissions tax in region 1. The assumption of constant returns to scale in production is important for this result.

[12] Böhringer et al. (2017) show that region 1 welfare can be improved by *marginally* increasing v^1 (from $v^1 = 0$) if $s^1 > 0$ and $t^1 > t^2$, but does not investigate analytically the optimal level of v^1, nor the effects of $v^1 = s^1$.

Corollary 1. *Assume the competitive equilibrium characterized by Lemma 1 with unilateral emission trading and 100% OBA as specified in Proposition 1. Then, a consumption tax* $v^1 = s^1$ *maximizes global welfare.*

Proof. See Appendix A.

Corollary 1 implies that also global welfare can be increased by coupling an existing domestic *OBA* regime with a domestic consumption tax equal to the *OBA* subsidy. For global welfare, terms-of-trade effects are of minor importance, and we will see that the numerical results (where terms-of-trade effects are important) are more in accordance with Corollary 1 than Proposition 1.

The carbon leakage targeted by the *OBA* policy depends crucially on the Armington elasticity $1/(1-\theta)$. Specifically, we show in Appendix A that the EITE good consumption ratio $\bar{f}^j / \bar{d}^j$ approaches $\beta/(1-\beta)$ if the Armington elasticity approaches perfect complements (i.e., as $\theta \to -\infty$).[13] Carbon leakage is clearly a moot point in this case. Indeed, *OBA* would increase foreign production of the EITE good if the Armington elasticity is sufficiently low, as increased output and hence consumption of the domestic EITE good will lead to increased consumption and hence production of the foreign EITE good, increasing foreign emissions. Combining *OBA* with a consumption tax would both offset the negative effects of *OBA* and ameliorate the environmental damage caused by foreign EITE goods sold in the home region (cf., Lemma 1). If, on the other hand, the Armington elasticity is high, such that *OBA* reduces carbon leakage and hence may have positive effect on domestic utility, we still know from Proposition 1 that a well-specified consumption tax will increase utility of region 1.

In practice, the Armington elasticity may be difficult to pin down (see the discussion in Section 1). In this case, a policy that combines *OBA* with a domestic consumption tax on EITE goods may provide a sort of insurance policy. The rationale is simply that one potential downside with *OBA*, i.e., the excessive domestic consumption of EITE goods, is attenuated by the consumption tax.

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[13] See Appendix A for the competitive equilibria in the limiting cases of perfect EITE good complements and perfect EITE good substitutes.

In the next section, we explore the properties of standalone *OBA* and *OBA* coupled with a consumption tax numerically. We focus on the case where the regulating region is the European Union (EU). This example is of interest, because the EU currently implements emission pricing with output-based allocation of free emission quotas to producers of EITE goods.

3. Numerical Analysis

3.1 Non-technical model summary

For our quantitative impact assessment of alternative unilateral climate policy designs, we adopt a standard multi-region multi-sector computable general equilibrium (CGE) model of global trade and energy use (see e.g. Böhringer et al. 2015, 2018). The strength of CGE models is their rigorous microeconomic foundation in Walrasian equilibrium theory, which accommodates the comprehensive welfare analysis of market supply and demand responses to policy shocks. For the sake of brevity, we confine ourselves to a brief non-technical summary of key model characteristics. A detailed algebraic description of the generic model is provided in Appendix B.

Our model features a representative agent in each region who receives income from three primary factors: labor, capital, and specific fossil fuel resources for coal, natural gas, and crude oil. Labor and capital are inter-sectorally mobile within a region but immobile between regions. Fossil resources are specific to fossil fuel production sectors in each region.

All commodities except for fossil fuels are produced according to a four-level nested CES cost function combining inputs of capital (K), labor (L), energy (E), and material (M) – see Figure 1.

At the top level, a material composite trades off with an aggregate of capital, labor, and energy. At the second level, the material composite splits into non-energy intermediate goods whereas the aggregate of capital, labor and energy splits into a value-added component and the energy component. At the third level, capital and labor inputs enter the value-added composite subject to a constant elasticity of substitution; likewise, within the energy aggregate, electricity trades off with the composite of fossil fuels (coal, natural gas, and refined oil). At the fourth level, a CES function describes the substitution possibilities between coal, refined oil, and natural gas.

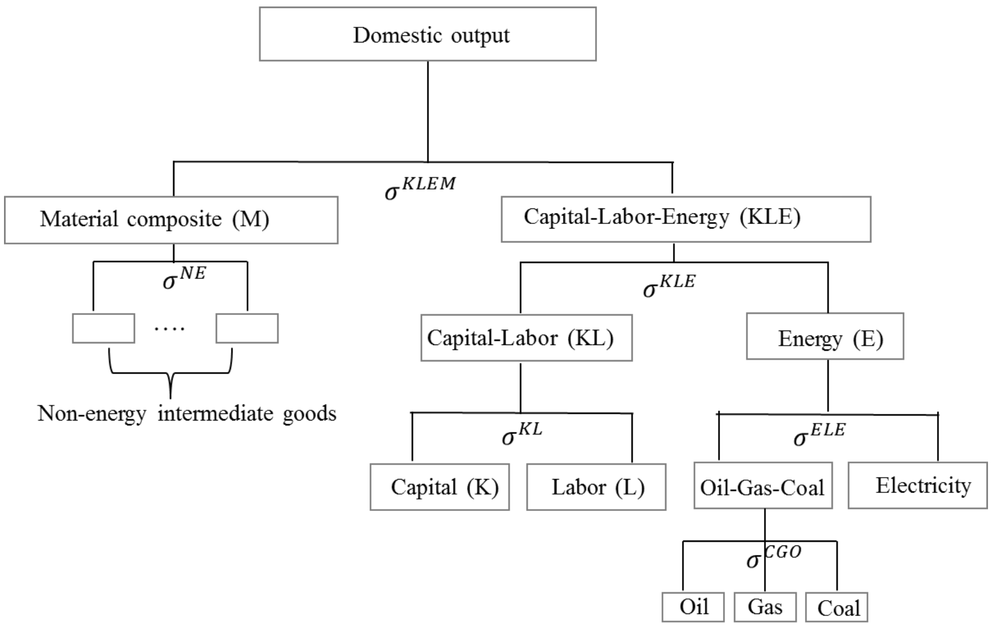

Figure 1. Production structure (see Appendix B for notations)

Fossil fuel production is represented by a constant-elasticity-of-substitution (CES) cost function, where the demand for the specific resource trades off with a Leontief composite of all other inputs.

Final consumption demand in each region is determined by the representative agent who maximizes welfare subject to a budget constraint with fixed investment and exogenous government provision of public goods and services. Consumption demand of the representative agent is given as a CES composite that combines consumption of composite energy and a CES aggregate of other consumption good. Substitution possibilities across different energy inputs in consumption are depicted in a similar nested CES structure as with production.

Bilateral trade is modeled following Armington's differentiated goods approach, where domestic and foreign goods are distinguished by origin (Armington, 1969). A balance of payment constraint incorporates the base-year trade deficit or surplus for each region.

CO_2 emissions are linked in fixed proportions to the use of coal, refined oil and natural gas, with CO_2 coefficients differentiated by fuels and sector of use. Restrictions to the use of CO_2 emissions in production and consumption are implemented through explicit emission pricing of the carbon associated with fuel combustion either via CO_2 taxes or the auctioning of CO_2 emission allowances. CO_2 emissions abatement takes place by fuel switching (interfuel substitution) or energy savings (either by fuel-non-fuel substitution or by a scale reduction of production and final consumption activities).

3.2 Data and parametrization

For model parameterization, we use the most recent data from the Global Trade, Assistance and Production Project (GTAP –version 9) which includes detailed balanced accounts of production, consumption, bilateral trade flows as well as data on physical energy consumption and CO_2 emissions for the base-year 2011 in 140 regions and 57 sectors (Aguiar et al., 2016). As is customary in applied general equilibrium analysis, base-year data together with exogenous elasticities determine the free parameters of the functional forms. Elasticities in international trade (Armington elasticities) as well as factor substitution elasticities are directly provided by the GTAP database. The elasticities of substitution in fossil fuel sectors are calibrated to match exogenous estimates of fossil-fuel supply elasticities (Graham et al. 1999, Krichene 2002, Ringlund et al. 2008).

The GTAP dataset can be flexibly aggregated across sectors and regions to reflect specific requirements of the policy issue under investigation. As to sectoral disaggregation our aggregate dataset explicitly includes different primary and secondary energy carriers: Coal, Crude Oil, Natural Gas, Refined Oil, and Electricity. This disaggregation is essential in order to distinguish energy goods by CO_2 intensity and the degree of substitutability. In addition, we keep those GTAP sectors explicit in the aggregate dataset which are considered as emission-intensive and trade-exposed (EITE) industries such as Chemical Products, Non-Metallic Minerals, Iron & Steel, Non-Ferrous Metals, and Refined Oil, as well as the three transport sectors (Air Transport, Water Transport, and Other Transport). Following the EU ETS, all sectors except Electricity, Water Transport, Other Transport and Other Goods and Services are potentially entitled to free allocation (see Section 3.3).

Regarding regional coverage, we single out the EU and its eight most important trading partners as individual regions. The remaining countries are divided into three composite regions. Table 2 summarizes the sectors (commodities) and regions present in our model simulations.

A key parameter regarding the extent of leakage is the Armington elasticity, which determines the ease of substitution between domestically produced goods and goods produced abroad. The higher this elasticity, the more pronounced leakage becomes, as higher costs of domestic production to a larger degree will cause relocation of production. The size of the Armington elasticity will likely vary across sectors and regions. The elasticities are of course not possible to observe, and also hard to assess although some attempts have been done (e.g., Saito, 2004; Welsch, 2008). The GTAP database provides sector-specific estimates of the Armington elasticities (which are equal across regions). These estimates are however quite uncertain, and hence leakage exposure of different sectors is also uncertain. This is probably a main reason why a large group of sectors is deemed “highly exposed to leakage” in the EU ETS, leading to “substantial overcompensation” according to Martin et al. (2014).

To reflect this uncertainty, we construct probability distributions for the Armington elasticities (see Appendix C for details), and then perform Monte Carlo simulations. For each simulation (1000 in total), we make a draw from the probability distribution for all the OBA sectors. Then we run all policy scenarios (see next subsection) given this set of Armington elasticities.

A relevant question is whether the Armington elasticities in different sectors are correlated or not. In the main simulations, we consider that the Armington elasticities in different sectors are stochastically independent. In the sensitivity analysis, we also consider the opposite case, that is, the Armington elasticities in different sectors are perfectly correlated. In both variants, the Armington elasticities are equal across regions.

Table 2. Sectors and regions in the CGE model (acronyms provided in brackets)

Sectors and commodities	*Countries and regions*
Primary Energy	Europe – EU-28 plus EFTA (EUR)
Coal (COA)	United States of America (USA)
Crude Oil (CRU)	Japan (JPN)
Natural Gas (GAS)	Russia (RUS)
*Emission-intensive and trade-exposed sectors**	China (CHN)
Chemical Products (CRP)	India (IND)
Non-Metallic Minerals (NMM)	Brazil (BRA)
Iron and Steel (I_S)	Turkey (TUR)
Non-Ferrous Metals (NFM)	South Korea (KOR)
Refined Oil (OIL)	Other OECD (OEC)
Paper Products, Publishing (PPP)	OPEC (OPC)
Machinery and Equipment (OME)	Rest of the World (ROW)
Food Products (OFD)	
Beverages and Tobacco Products (B_T)	
Air Transport (ATP)	
Other ETS sectors (RES)	
Other sectors	
Electricity (ELE)	
Water Transport (WTP)	
Other Transport (OTP)	
Other Goods and Services (ROI)	

* Sectors that are entitled to output-based allocation in the main simulations – referred to as "OBA goods" in Table 3.

3.3 Scenarios

We consider the same policy scenarios as in the theoretical analysis (cf. Table 1 in Section 2), but now in the context of the EU. Our reference scenario (*REF*) is a situation where the EU implements economy-wide uniform emission pricing to reduce its emission by 20 percent of the base-year emissions.[14] We then quantify how the *REF* outcome changes if the region adopts in addition either output-based allocation (*OBA*), or *OBA* combined with a consumption tax (*CTAX*), cf. Table 3. In both cases, the additional policies are directed towards goods that are more or less emission-intensive and trade-exposed (referred to as "OBA goods"). In the main simulations, we follow the current situation in the EU ETS where a large group of sectors receive free allowances in proportion to output. In the sensitivity analysis, we consider the case where only the most emission-intensive and trade-exposed (EITE) goods are given free allowances. In the *OBA* and *CTAX* cases, we assume 100% allocation.[15] In the *CTAX* case we first consider a variety of tax rates to check whether the analytical result carries over. That is, according to Proposition 1 and Corollary 1, the optimal consumption tax is equal to the implicit output subsidy of the *OBA* (referred to as "*100% CTAX*"), both from a regional and global welfare perspective. Subsequently, we focus on the *100% CTAX* case. The consumption tax is applied to both final consumption and intermediate use of OBA goods.

Table 3. Policy scenarios for the EU*

REF	Economy-wide emission price
OBA	Output-based allocation to "OBA goods"
CTAX	Output-based allocation + consumption tax for "OBA goods"

* See Table 2 and the text for definition of "OBA goods"

As mentioned before, in order to avoid explicit damage valuation from greenhouse gas emissions, we keep the global emissions constant across the three policy scenarios. This means that the EU adjusts its unilateral emission constraint so that the same global emission cap is reached. The cap is set equal to the global emissions in the *REF* scenario. As the two alternative policy scenarios turn out to reduce leakage

[14] Uniform emission pricing to achieve some emission reduction target can either be implemented through an emission tax which is set at a sufficiently high level or equivalently through an emissions cap-and-trade system.
[15] By 100% allocation, we mean that in a given scenario the number of free allowances given to the OBA sectors is equal to the (endogenous) emissions in these sectors. The implicit output subsidy of *OBA* is equal to the value of the free allowances per unit of production

compared to *REF* (see next subsection), the emission constraint in the EU will be slightly less stringent in *OBA* and *CTAX* than in *REF*.

3.4 Results

We start by looking at welfare effects (measured in terms of Hicksian equivalent variation of income) for the EU. The *REF* scenario involves an economy-wide CO_2 price of 106 USD per ton (on average). When implementing output-based allocation (*OBA*), and adjusting the EU cap to keep global emissions unchanged, welfare in the EU decreases by on average 0.16% vis-à-vis *REF*.

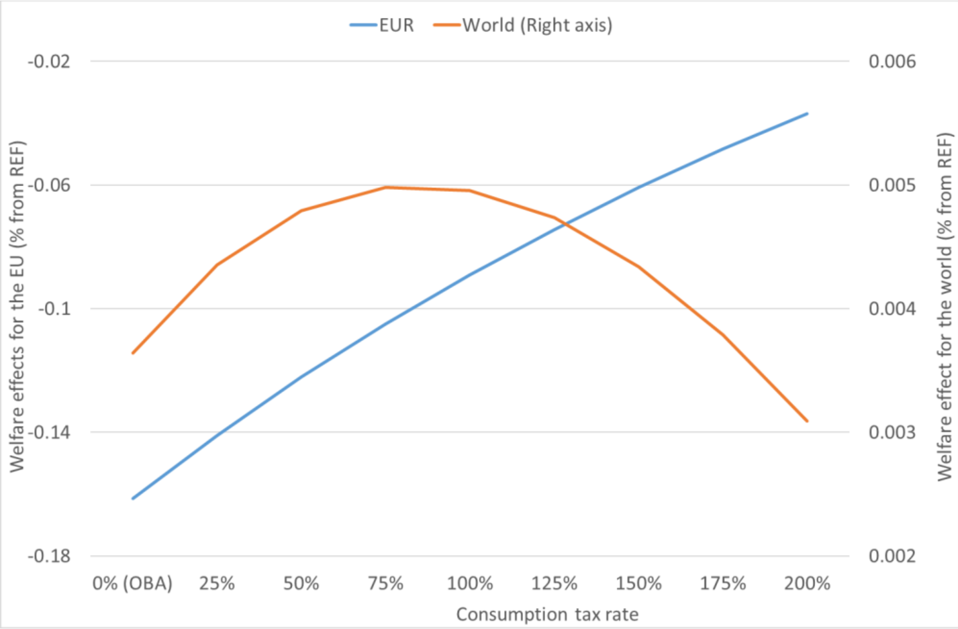

Figure 2. Welfare effects in the EU and the world vis-à-vis *REF*, for different consumption tax rates in the EU (in %). Average results based on 1000 runs

Remember that *OBA* has four important effects: First, it reduces leakage, which is welfare-improving as it relaxes EU's own emission cap. Second, it involves subsidizing foreign consumption of the OBA goods, which is a negative side effect. Both these effects are bigger the more leakage exposed the sectors are. Third, *OBA* stimulates too much use of the OBA goods domestically, which has a negative welfare effect. The less leakage exposed the sectors are, the more important this third effect is. Fourth, *OBA* has terms-of-trade effects, which in general can be either positive or negative depending on the trade pattern. As the EU is a net exporter of

OBA goods, and output-based allocation tends to reduce the price of these goods, the terms-of-trade effects are likely negative for the EU.[16] Thus, there is one positive and three negative effects of *OBA*, and the simulations suggest that the net effect is negative.

When also implementing the consumption tax (*CTAX*), we see from Figure 2 that EU's welfare on average improves monotonically as the *CTAX* rate is increased. That is, in the numerical simulations, the optimal consumption tax rate for the EU is far higher than 100%,[17] which according to Proposition 1 should be the optimal tax rate. The explanation for this is again terms-of-trade effects, which were absent in the theoretical analysis.[18] Other regions are on average worse off when the consumption tax is imposed in the EU.

According to Corollary 1, the optimal consumption tax rate is 100% also when considering global welfare. This is quite consistent with our numerical results, which indicate that the optimal tax rate from a global perspective is on average 80-85% of the OBA-rate. At the global level, terms-of-trade effects are negligible since terms-of-trade benefits for one region are terms-of-trade losses for another region. When looking more closely at the results, we find that the optimal consumption tax (from a global perspective) tends to increase with the Armington elasticity. For low elasticities, the optimal tax rate is slightly above 100% of the OBA-rate.

Thus, we may conclude that if the EU were to choose a consumption tax that is beneficial both for the EU and for the world in aggregate, a tax of about the same order as the OBA-rate would be appropriate.

Next, we want to focus on the *100% CTAX* variant, and compare it with *OBA*, which is the current policy in the EU. We are interested in whether *100% CTAX* is always an

[16] Other regions are on aggregate better off when the EU implements *OBA*, which confirms the terms-of-trade detoriation for the EU.

[17] The optimal consumption tax rate for the EU is in the range 850-900% of the *OBA*-rate. This may sound like a very high tax rate, but note that a 100% consumption tax amounts to less than 2.5% increase in the price of the different OBA goods (except Air Transport, for which the price increase is 8%).

[18] If we search for the optimal consumption tax rate *in the absence of OBA*, it is in the range 600-650%. Thus, from a regional point of view, a quite substantial consumption tax is beneficial, mostly due to terms-of-trade effects. Further, we observe that when *OBA* is implemented, the optimal consumption tax rate increases by around 250%-points (from 600-650% to 850-900%).

improvement vis-à-vis *OBA*, i.e., irrespective of whether the leakage exposure (Armington elasticities) is high or low. The results are shown in Figures 3 (EU welfare) and 4 (global welfare).

From Figure 3 we see that the consumption tax improves EU welfare vis-à-vis *OBA* in all simulations – on average by 0.07% (cf. also Figure 2). Thus, the results suggest that implementing a consumption tax in addition to output-based allocation is smart hedging against carbon leakage. The consumption tax mitigates the third effect of *OBA* mentioned above, i.e., too much use of the OBA goods domestically. In addition comes the beneficial terms-of-trade effects noted above.

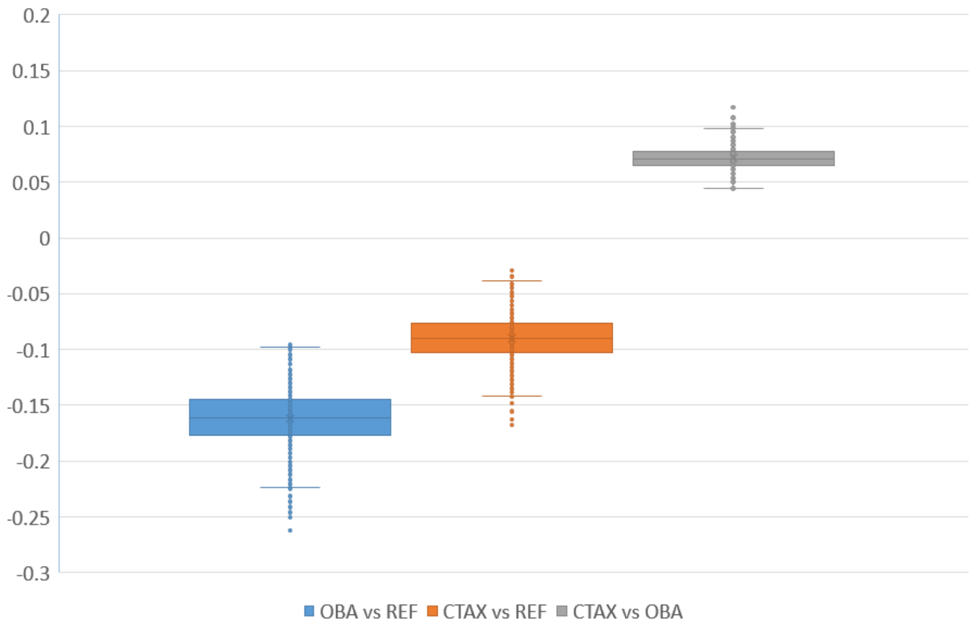

Figure 3. Welfare effects in the EU vis-à-vis *REF*, and *100% CTAX* vis-à-vis *OBA* (in %). Box-and-Whisker plot based on 1000 runs[19]

Figure 4 shows that the consumption tax (*100% CTAX*) also improves global welfare vis-à-vis *OBA* in almost all simulations (966 of 1000 runs). Thus, even when disregarding terms-of-trade effects, the consumption tax may be regarded as smart hedging against carbon leakage.

[19] The Box-and-Whisker plot shows minimum, first quartile, median, third quartile, and maximum.

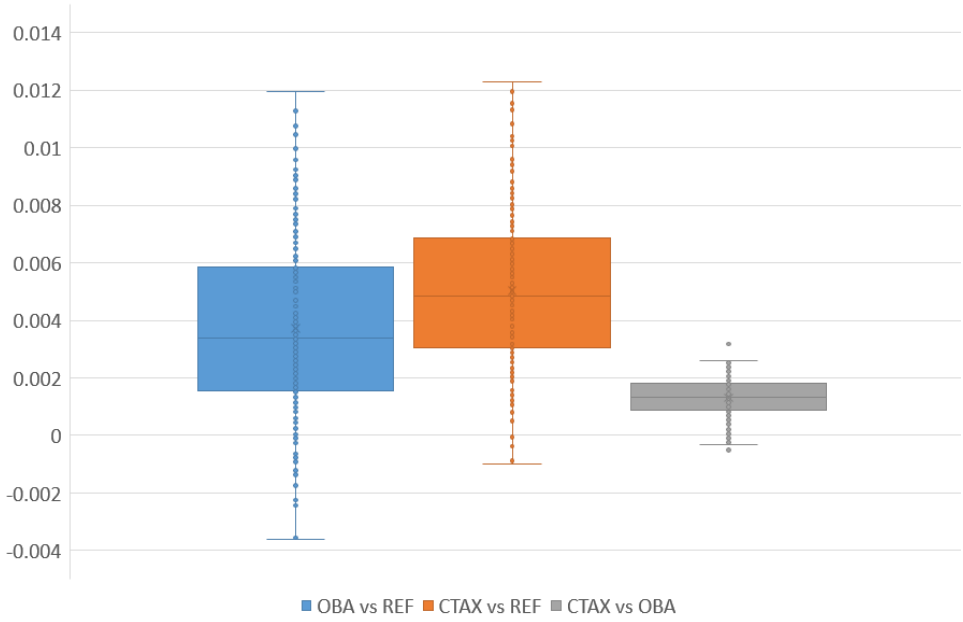

Figure 4. Global welfare effects vis-à-vis *REF*, and *100% CTAX* vis-à-vis *OBA* (in %). Box-and-Whisker plot based on 1000 runs

As pointed out before, the less leakage exposed OBA goods are, the more likely it is that the effects of *OBA* are negative. Further, the more beneficial it would be to supplement *OBA* with a consumption tax. This is confirmed in our simulations, see Figure 5. The figure shows how EU and global welfare gains from the consumption tax (i.e., *100% CTAX* vs. *OBA*) vary with the weighted average Armington elasticity of the OBA goods.[20] We notice that the consumption tax has bigger welfare gains when the Armington elasticity is low. As Armington elasticities can be seen as a proxy for leakage exposure, we conclude that the less leakage exposed the sectors are, the more important it is to correct the undesired effects of output-based allocation, both from a regional and global perspective.

[20] The weights used are the production value of the sectors.

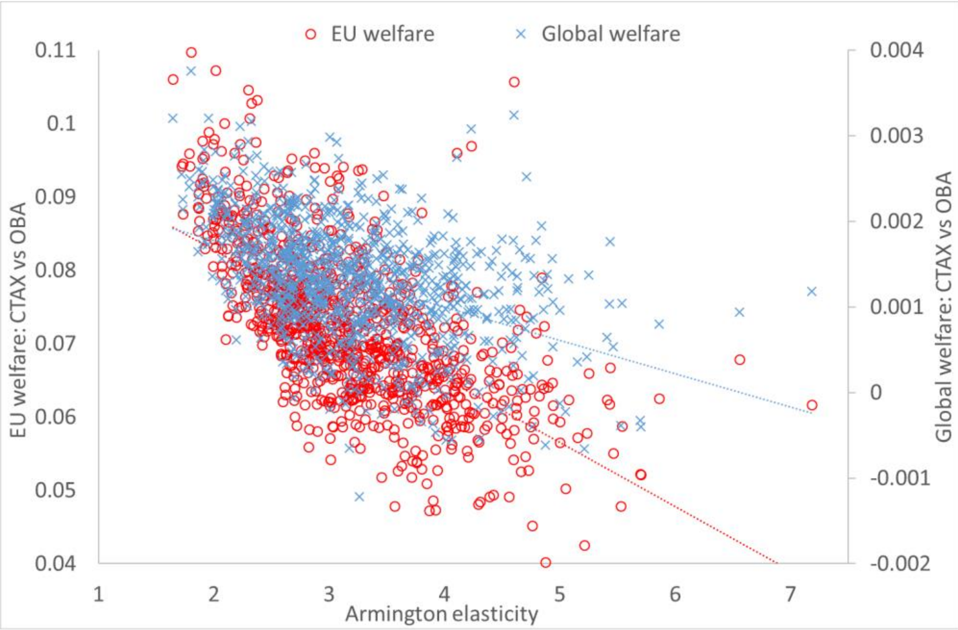

Figure 5. Relationship between Armington elasticity and welfare gain from *100% CTAX* vis-à-vis *OBA* (in %). Scatter plot based on 1000 runs

Although the consumption tax may be regarded as smart hedging against leakage, it doesn't mean that leakage is reduced. In fact, the leakage rate is 1 percentage point *higher* in *100% CTAX* than in *OBA*. This may seem surprising at first – after all the consumption tax reduces demand for OBA goods, which are typically emission-intensive and trade-exposed. The explanation is that the consumption tax not only reduces consumption of OBA goods in the EU – it also shifts to some degree market shares from the EU to non-EU regions. In fact, overall output of OBA goods outside the EU increases slightly. The reason is that the consumption tax not only applies to end-use of OBA goods, but also to intermediate use of these goods. As many OBA sectors use various OBA goods as inputs in their production, their costs of production increase when this tax is introduced. This makes domestic production of OBA goods slightly less competitive, and shifts production to some degree out of the EU. As one motivation for allocating allowances, in addition to mitigating leakage, is to prevent losses in competitiveness, this may be regarded as a undesirable implication of the consumption tax.

We can further investigate the competitiveness implications, by examining the effects on net exports in the three scenarios across three important manufacturing industries, that is, Iron & Steel (I_S), Non-Metallic Minerals (NMM), and Chemical Products (CRP), see Figure 6. We see that carbon pricing alone reduces net export as production is relocated outside Europe – as expected. The biggest effects, measured in monetary values, are seen for Chemical Products. *OBA* mitigates the loss in competitiveness, but net export is still negative (vis-à-vis *BaU*) for all three sectors. On average, the reduction in net export is about halved when *OBA* is implemented. With the consumption tax, net export drops again, but is slightly closer to the *OBA* outcome than the *REF* outcome. Note however that the reduced net export from the consumption tax amounts to less than 0.5% of EU production of these goods.

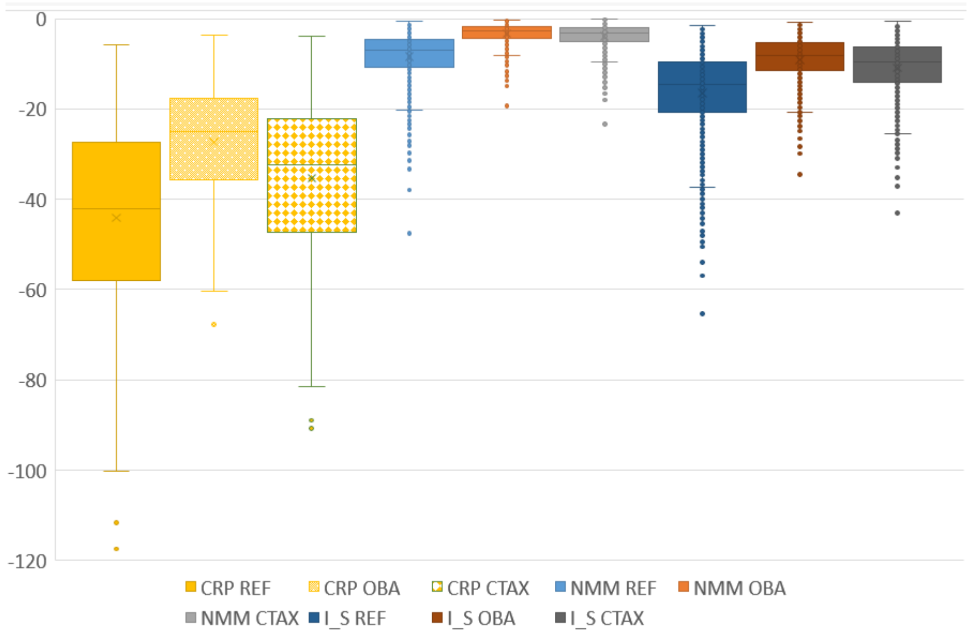

Figure 6. Effects on net trade (export minus import) in the EU of four EITE products. Changes vis-à-vis BaU (billion USD). Box-and-Whisker plot based on 1000 runs

3.5 Sensitivity analysis

We examine the sensitivity of our results along different dimensions, where we focus on the welfare effects of imposing a consumption tax in a situation where an ETS is already in place together with output-based allocation to the same sectors as before (i.e., *100% CTAX* vs *OBA*). Figure 7 considers the case where the policy region differs. We notice that if China or the US is the policy region, implementing a consumption tax is (almost) always beneficial (both for the policy region and for the world in aggregate), but the benefits are smaller than in the EU case. If all three regions have implemented ETS with *OBA* (but with different CO2-prices), imposing a consumption tax in all regions is again beneficial and the aggregate effects for the three regions are slightly higher than the weighted average of the single region benefits. Thus, the more regions are implementing carbon pricing jointly with *OBA*, the more beneficial it is to also impose the consumption tax.

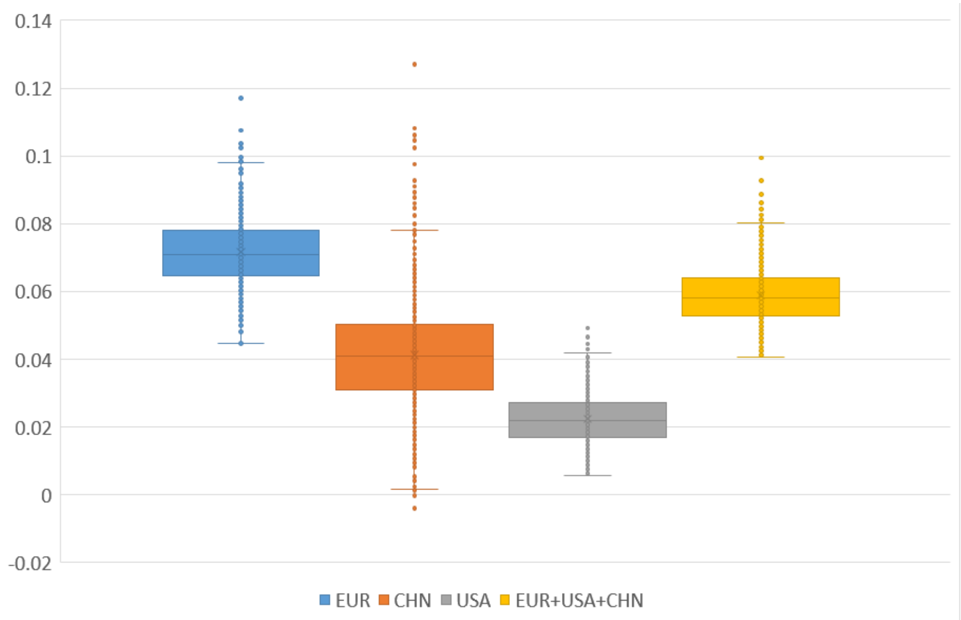

Figure 7. Regional welfare effects of *100% CTAX* vis-à-vis *OBA* (in %). Box-and-Whisker plot based on 1000 runs

Next, we consider alternative assumptions about the size of the emission reduction in the EU. If the EU reduces emissions by 30% instead of 20%, the benefits of the consumption tax increases by about 50% on average, cf. Figure 8. Furthermore, if a very ambitious climate policy is introduced in the EU, reducing emissions by 50%, the welfare gains from the consumption tax triple (compared to the base case of 20% reduction). In both cases, the consumption tax enhances welfare in all the runs.

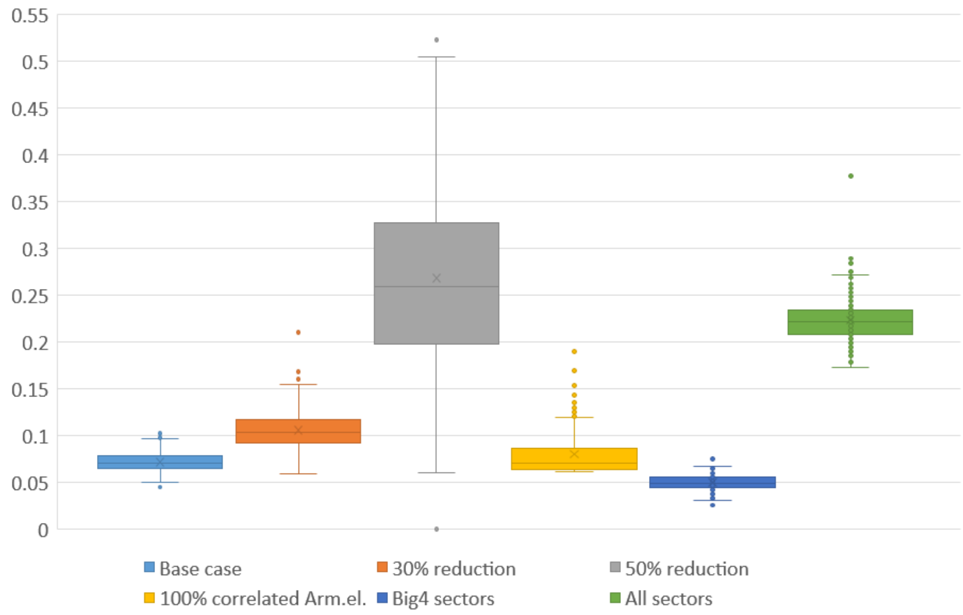

Figure 8. Welfare effects in the EU of *100% CTAX* vis-à-vis *OBA* (in %). Box-and-Whisker plot based on 1000 runs

If output-based allocation is only provided to the four big EITE sectors Iron & Steel, Non-Metallic Minerals, Chemical Products, and Refined Oil, the consumption tax is still increasing welfare for the EU, but the benefits are somewhat reduced. On the other hand, if *OBA* were provided to all sectors of the ETS, including the electricity sector, the consumption tax would become quite desirable as it reduces the too high consumption of electricity.

Finally, we notice that to what degree the Armington elasticities are correlated across sectors has fairly limited importance for the welfare effect of the consumption tax. The average welfare benefit (across the Monte Carlo simulations) is almost the same in the two extreme cases (i.e., no correlation and 100% correlation).[21]

4. Concluding remarks

Despite the Paris Agreement on coordinated action to mitigate climate change, the stringency of climate policies differs quite substantially between countries, and will likely continue to do so in the future. This can result in carbon leakage associated with

[21] We have also tested the effects of different fossil fuel elasticities. The results are fairly similar to the base case results and are thus not shown in the figure.

the relocation of emission-intensive and trade-exposed (EITE) industries from countries with more stringent climate policies to countries with laxer regulations. To reduce the extent of such leakage, a common approach is to supplement an emission trading system with free allocation of allowances proportional to the output of exposed industries, so-called output-based allocation (OBA). In the EU ETS, OBA has been in place since 2013, and will continue also after 2020.

A disadvantage of granting OBA to EITE goods is that it tends to stimulate too much use of these goods, as the allocation works as an implicit output subsidy. Substitution towards less emission-intensive goods is hence restrained. In this paper we have analyzed the impacts of adding a consumption tax on all use of the EITE goods. We have shown analytically that under certain conditions it is optimal from both a regional and global welfare perspective to impose a consumption tax that is equivalent in value to the OBA-rate already in place.

The theoretical result is confirmed in the context of the EU ETS, when using a multi-region and multi-sector computable general equilibrium model of global trade and energy use calibrated to empirical data. We show that the addition of sector-specific consumption taxes increases EU welfare, irrespective of how leakage exposed the sectors actually are. Martin et al. (2014) have found that there has been substantial overallocation of allowances in the EU ETS for the given carbon leakage risk. Our results suggest that the climate policy becomes more cost-effective with respect to uncertainties about leakage exposure when adding consumption taxes. The potentially distortive effects of allowance overallocation – by including too many sectors with limited carbon leakage risk or warranting too generous allocation – are attenuated. Additional administrative costs of implementing consumption taxes are likely to be moderate, as the size of the tax of a specific good should correspond to the OBA-rate for this particular good, i.e., information that is already there. We thus conclude that supplementing output-based allocation with consumption taxes constitutes smart hedging against carbon leakage.

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Appendix A: Analytical proofs and derivations

Proof of Lemma 1: Here we also prove that $p^{x1} = p^{x2} = c^x \equiv p^x$ (which was assumed in the formulation of (6)). Hence, we replace the producers maximization problem in (6) with $\pi^{xj} = \arg\max_{x^{1j},x^{2j}} \left(p^{x1}x^{1j} + p^{x2}x^{2j} - c^x\left(x^j\right)\right)$ ($j = 1,2$). The firms' first order conditions for profit maximization are:

$$
\begin{aligned}
&p^{x1} = p^{x2} = p^x = c^x, \\
&p^{zj} = c^z + \xi^z \phi^z a^{zj}, \quad j = 1,2, \\
&p^{d1} + s^1 = p^{d2} + s^1 = c^d + \xi^y \phi^y a^d, \\
&p^{f1} = p^{f2} = c^f + \xi^y \phi^y a^f, \\
&\phi^z a^{zj} = t^j, \phi^y a^d = t^1, \phi^y a^f = t^2, \quad j = 1,2,
\end{aligned} \tag{18}
$$

Here $a^g = \xi^g g - e^g$, so the equation in the bottom row states that the emission price equals the cost of marginal emission reductions. We have $a^{z1} = \xi^z z^1 + e^f + e^d + e^{z2} - \overline{E}$, and similarly for the other goods. Note that the market equilibrium for the x^j good requires that $x^j = \overline{x}^{1j} + \overline{x}^{2j}$; i.e., the volume of x produced in region j must equal the sum of consumption of good x originating from region j in both regions. This implies the market equilibrium condition for good x in (3). We also observe that the firms' profits π^{xj} in (6) are concave in production in equilibrium, given the demand functions associated with the consumer utility maximization problem (7). The model does not uniquely determine production of x, however, only that we must have $x^1 + x^2 = \overline{x}^1 + \overline{x}^2$. This does not matter for the results, because there are no emissions or profits associated with production of x.

The representative consumers' maximize utility (1) subject to (2) and the budget constraint (8). The Lagrangian is:

$$
L^j = u^j\left(\overline{x}^j, \overline{y}^j, \overline{z}^j\right) + \lambda^j\left(m^j + A^j - p^{xj}\overline{x}^j - p^{zj}\overline{z}^j - \left(p^{dj} + v^j\right)\overline{d}^j - \left(p^{fj} + v^j\right)\overline{f}^j\right), \quad j = 1,2. \tag{19}
$$

The CES utility function is strictly concave in the decision variables under our assumptions that $\beta^j, \alpha^{xj}, \alpha^{yj}, \alpha^{zj} > 0$ and $\rho, \theta < 1$. Hence, the Lagrangian (19) is strictly concave, being a sum of strictly concave and weakly concave functions. The first order conditions associated with (19) simplify to:

$$
\begin{aligned}
&\frac{p^{xj}}{p^{zj}} = \frac{\alpha^{xj}}{\alpha^{zj}} \left(\frac{\overline{z}^{j}}{\overline{x}^{j}} \right)^{1-\rho}, \quad j = 1,2, \\
&\frac{p^{dj} + v^{j}}{p^{fj} + v^{j}} = \frac{\beta^{j}}{1-\beta^{j}} \left(\frac{\overline{f}^{j}}{\overline{d}^{j}} \right)^{1-\rho}, \quad j = 1,2, \\
&\frac{p^{xj}}{p^{dj} + v^{j}} = \frac{1}{\beta^{j}} \frac{\alpha^{xj}}{\alpha^{yj}} \frac{\left(\overline{d}^{j}\right)^{1-\theta}}{\left(\overline{x}^{j}\right)^{1-\rho}} \left(\beta \left(\overline{d}^{j}\right)^{\theta} + (1-\beta)\left(\overline{f}^{j}\right)^{\theta} \right)^{1-\frac{\rho}{\theta}}, \quad j = 1,2.
\end{aligned}
\tag{20}
$$

Together with the budget constraint (8), (20) constitutes a system of four equations with four unknowns (for each *j*). The first order conditions for the firms (18) and the representative consumer (20) yields equations (9) to (12) in Lemma 1.

The consumer's budget constraint in competitive equilibrium (including lump sum transfers of net taxes and profits treated as exogenous by the representative consumer) is given by equation (8). Profits from *x* is $p^{x}x^{j} - c^{x}x^{j} = 0$ (cf., (6) and (18)) and omitted from the calculation of the budget constraints. For region 1, we have:

$$
\begin{aligned}
A^{1} &= \sum\nolimits_{g} t^{1}e^{g1} + v^{1}\left(\overline{d}^{1} + \overline{f}^{1}\right) - s^{1}d + \sum\nolimits_{g} \pi^{g1} \\
&= t^{1}\left(e^{z1} + e^{d}\right) + v^{1}\left(\overline{d}^{1} + \overline{f}^{1}\right) - s^{1}\left(\overline{d}^{1} + \overline{d}^{2}\right) + p^{z1}\overline{z}^{1} - c^{z}\left(z^{1}, e^{z1}\right) - t^{1}e^{z1} + \left(p^{d1} + s^{1}\right)\overline{d}^{1} + \left(p^{d2} + s^{1}\right)\overline{d}^{2} - c^{d}\left(d^{1}, e^{d}\right) - t^{1}e^{d} \\
&= v^{1}\left(\overline{d}^{1} + \overline{f}^{1}\right) + \left(p^{z1}\overline{z}^{1} - c^{z}\left(z^{1}, e^{z1}\right)\right) + \left(p^{d1}\overline{d}^{1} + p^{d2}\overline{d}^{2} - c^{d}\left(d^{1}, e^{d}\right)\right).
\end{aligned}
$$

Inserting in the budget constraint of region 1 (8), we have:

$$
\begin{aligned}
& m^{1} + v^{1}\left(\overline{d}^{1} + \overline{f}^{1}\right) + p^{z1}\overline{z}^{1} - c^{z}\left(z^{1}, e^{z1}\right) + p^{d1}\overline{d}^{1} + p^{d2}\overline{d}^{2} - c^{d}\left(d^{1}, e^{d}\right) \geq p^{x1}\overline{x}^{1} + p^{z1}\overline{z}^{1} + \left(p^{d1} + v^{1}\right)\overline{d}^{1} + \left(p^{f1} + v^{1}\right)\overline{f}^{1} \\
\Leftrightarrow\ & m^{1} + p^{d2}\overline{d}^{2} - p^{f1}\overline{f}^{1} - c^{x}\overline{x}^{1} - c^{d}\left(d^{1}, e^{d}\right) - c^{z}\left(z^{1}, e^{z1}\right) \geq 0 \\
\Leftrightarrow\ & m^{1} + \left(c^{d} + \xi^{y}t^{1} - s^{1}\right)\overline{d}^{2} - c^{f}\overline{f}^{1} - c^{x}\overline{x}^{1} - c^{d}\left(d^{1}, e^{d}\right) - c^{z}\left(z^{1}, e^{z1}\right) \geq 0,
\end{aligned}
$$

which is equivalent with the budget constraint for region 1 (13) (we used (18) in the equivalences). The derivation of the budget constraint for region 2 is similar and not repeated here. We observe that the budget constraints must hold with equality in equilibrium (for finite m^{j}), because utility can always be increased by more consumption of one or more goods (cf., the utility function (1)). This proves that equations (9) to (13) in Lemma 1 are necessary conditions for optimum. These conditions are also sufficient, because the second order conditions are fulfilled for firms (profits are concave in production) and the representative consumer (the Lagrangian is concave in consumption of the four goods).

Lemma 1 in the limit cases of perfect EITE good compliments and substitutes:

With perfect compliments, $\theta \to -\infty$, equation (2) becomes $y = \min\left(\beta \overline{d}^{j}, (1-\beta)\overline{f}^{j}\right)$, which is maximized if $\beta \overline{d}^{j} = (1-\beta)\overline{f}^{j}$. Utility from the EITE good is $\beta \overline{d}^{j}$ (or, equivalently, $(1-\beta)\overline{f}^{j}$). Hence, we must have $\beta \overline{d}^{j} / (1-\beta) = \overline{f}^{j}$ in optimum. The representative consumer's Lagrangian is:

$$L^{PC,j} = \left(\alpha^{xj}\left(\overline{x}^{j}\right)^{\rho} + \alpha^{yj}\left(\beta \overline{d}^{j}\right)^{\rho} + \alpha^{zj}\left(\overline{z}^{j}\right)^{\rho}\right)^{\frac{1}{\rho}} + \lambda^{j}\left(m^{j} + A^{j} - p^{xj}\overline{x}^{j} - p^{zj}\overline{z}^{j} - \left(\left(p^{dj} + v^{j}\right) - \left(p^{fj} + v^{j}\right)\frac{\beta}{1-\beta}\right)\overline{d}^{j}\right).$$

and the competitive equilibrium is characterized by Lemma 1 with equations (10) and (11) replaced by:

$$\overline{f}^{j} = \frac{\beta^{j}}{1-\beta^{j}}\overline{d}^{j},$$

$$\frac{c^{x}}{c^{d} + \xi^{y} t^{1} - s^{1} + v^{j}} = \frac{\alpha^{xj}}{\alpha^{yj}} \frac{1}{\left(\beta^{j}\right)^{\rho}} \left(\frac{\overline{d}^{j}}{\overline{x}^{j}}\right)^{1-\rho}.$$

With perfect substitutes, $\theta \to 1$, equation (2) becomes $\beta \overline{d}^{j} + (1-\beta)\overline{f}^{j}$. Demand for *d* (*f*) is zero if $\left(1-\beta^{j}\right)/\beta^{j} < (>) P^{dj} / \mathrm{P}^{fj}$. Hence, a corner solution is likely to occur in competitive equilibrium with constant returns to scale in production. There also exists a continuum of interior solutions if $\left(1-\beta^{j}\right)/\beta^{j} = P^{dj} / \mathrm{P}^{fj}$. We do not solve the model in the limiting case with perfect substitutes here; see Böhringer et al. (2017) for an analysis of output-based rebating and EITE good consumption taxes under the assumption of perfect substitute EITE goods (with convex production costs).

Proof of Proposition 1 and Corollary 1: We first observe that all equations in Lemma 1 are equal under *OBA* and *CTAX* for region 2, and that foreign consumption is unaffected by v^{1} (this hinges on constant returns to scale in production). We therefore let exports x^{21} and d^{2} be treated as constants (determined by Lemma 1 with

CTAX/OBA) in this proof.[22] Further, regarding *x*, the cost of consuming *x* in region 1 is $c^x x^{11} + p^{x1} x^{21} = c^x \overline{x}^1$, (cf., (18) and (3)). Hence, $c^x \overline{x}^1$ is the cost of consuming *x* in the social planner's Lagrangian L^{SP} below.

Assume that a social planner maximizes region 1 welfare, given the utility function (1), the exogenous global emissions cap (4), the production cost (5) and the budget constraint (8). We constrain the social planner from exploiting market power and foreign firms are price takers. Further, the regulator knows that foreign emissions are unregulated and internalizes the increased abatement demanded to uphold the global emissions cap to compensate for foreign emissions associated with imports of $\overline{f}^1$. The social planner's Lagrangian is:

$$L^{SP} = u^1\left(\overline{x}^1, \overline{y}^1, \overline{z}^1\right) + \lambda\left(m^1 - c^x \overline{x}^1 - c^z\left(\mathrm{z}, e^{z1}\right) - c^d\left(d, e^d\right) + p^{d2} d^2 - c^f \overline{f}^1\right) + \mu\left(\overline{E} - e^{z1} - \zeta^z z^2 - e^d - \zeta^y f\right),$$

where λ and μ are the Lagrange multipliers associated with the budget constraint (8) and the global emissions cap (4), respectively. The Lagrangian is maximized w.r.t $\overline{x}^1$, $\overline{z}^1$, $\overline{d}^1$, $\overline{f}^1$, e^{z1} and e^d. The first order conditions imply:

$$\begin{aligned}
&\frac{c^x}{c^z + \varsigma\xi^z} = \frac{\alpha^{x1}}{\alpha^{z1}}\left(\frac{\overline{z}^1}{\overline{x}^1}\right)^{1-\rho},\\
&\frac{c^d + \varsigma\xi^y}{c^f + \varsigma\xi^y} = \frac{\beta^1}{1-\beta^1}\left(\frac{\overline{f}^1}{\overline{d}^1}\right)^{1-\rho},\\
&\frac{c^x}{c^d + \varsigma\xi^y} = \frac{\alpha^{x1}}{\alpha^{y1}}\frac{1}{\beta^1}\left(\beta^1\left(\overline{d}^1\right)^{\theta} + \left(1-\beta^1\right)\left(\overline{f}^1\right)^{\theta}\right)^{1-\frac{\rho}{\theta}}\frac{\left(\overline{d}^1\right)^{1-\theta}}{\left(\overline{x}^1\right)^{1-\rho}},\\
&\varsigma \equiv \frac{\mu}{\lambda} = \phi^z a^{z1} = \phi^y a^d.
\end{aligned} \tag{21}$$

The necessary conditions (21) are, together with (4) and (13), also sufficient, because the Lagrangian L^{SP} is concave in the decision variables. Together with the budget

---

[22] Whereas restricting x^{21} to be constant does not matter for the zero emission and zero profit good *x (*profits from exports of *x* is $p^{x2}x^{12} - c^x x^{12} = 0$ *)*, the social planner solution would have the exports of domestic EITE good d^2 satisfy $p^{d2} = c^d + \varsigma\xi^y$ if given the opportunity (*OBA* and *CTAX* both feature $p^{d2} = c^d$, implying too much foreign consumption of the domestic EITE good).

constraint (13) (for $j=1$) and the global emission cap (4), (21) constitutes a system of 7 equations with 7 unknowns which solves social planners' problem.

The admissibility conditions (4) and (13) enter both in the social planners solution and the competitive equilibrium in Lemma 1. Further, the remaining equations (9) to (12) in Lemma 1 for $j=1$ are equal to (21) under *CTAX* regulation with $v^1 = s^1 = \zeta^y t^1$ (here we use that $\varsigma = t^{1,CTAX}$, because the endogenous ς and $t^{1,CTAX}$ enter identical fully determined systems of equations (except for production of *x*)). It follows that the Ctax regime described in Lemma 1 solves the social planner's problem. This proves Proposition 1. Corollary 1 follows immediately, because the allocation in region 2 is unaffected by the consumption tax v^1.

Appendix B: Algebraic summary of computable general equilibrium (CGE) model

We provide a compact algebraic description for the generic multi-region multi-sector CGE model underlying our quantitative simulation analysis. Tables B.1 – B.5 explain the notations for variables and parameters employed within our algebraic exposition. The algebraic summary is organized in three sections that state the three classes of economic equilibrium conditions constituting a competitive market outcome: zero-profit conditions for constant-returns-to-scale producers, market-clearance conditions for commodities and factors, and income balances for consumers. In equilibrium, these conditions determine the variables of the economic system: zero-profit conditions determine activity levels of production, market-clearance conditions determine the prices of goods and factors, and income-balance conditions determine the income levels of consumers. We use the notation Π_{ir}^{X} to denote the unit-profit function of production activity *i* in region *r* where *X* is the name assigned to the associated production activity.[23] For a condensed representation of market equilibrium conditions, we can differentiate the unit-profit functions with respect to input and output prices in order to obtain compensated demand and supply coefficients (Hotelling's lemma) which then enter the market equilibrium conditions. Numerically, the model is implemented in GAMS.[24]

Table B.1: Indices and sets

i (alias j)	Index for sectors and goods - including the composite private consumption good (*i*=*C*), the composite public consumption good (*i*=*G*), and the composite investment good (*i*=*I*)
r (alias s)	Index for regions
NE	Set of non-energy goods
FF	Set of primary fossil fuels: Coal, crude oil, gas
CGO	Set of fuels with CO_2 emissions: Coal, gas, refined oil

[23] Note that we can decompose production in multiple stages (nests) and refer to each nest as a separate sub-production activity. In our exposition below, we specify for example the choice of capital-labor inputs as a price-responsive sub-production: Π_{ir}^{KL} (*X*=*KL*) then denotes the zero-profit condition of value-added production in sector *i* and region *r*.

[24] The model code and data to replicate simulation results are readily available upon request.

Table B.2: Variables

	Activity levels
KL_{ir}	Value-added composite in sector *i* and region *r*
E_{ir}	Energy composite in sector *i* and region *r*
Y_{ir}	Production in sector *i* and region *r*
M_{ir}	Import composite for good *i* and region *r*
A_{ir}	Armington composite for good *i* in region *r*
	Price levels
p_{ir}^{KL}	Price of aggregate value-added in sector *i* and region *r*
p_{ir}^{E}	Price of aggregate energy in sector *i* and region *r*
p_{ir}^{Y}	Output price of good *i* produced in region *r*
p_{ir}^{M}	Import price aggregate for good *i* imported to region *r*
p_{ir}^{A}	Price of Armington good *i* in region *r*
w_r	Wage rate in region *r*
v_r	Price of capital services in region *r*
q_{ir}	Rent to natural resources in region *r* ($i \in FF$)
$p_r^{CO_2}$	CO_2 emission price in region *r*
	Income levels
INC_r	Income level of representative household in region r

Table B.3: Cost shares

θ_{ir}^{K}	Cost share of capital in value-added composite of sector *i* and region *r* ($i \notin FF$)
θ_{ir}^{ELE}	Cost share of electricity in energy composite in sector *i* in region *r* ($i \notin FF$)
θ_{jir}^{CGO}	Cost share of fuel *j* in the fuel composite of sector *i* in region *r*($i \notin FF$), ($j \in CGO$)
θ_{ir}^{KLE}	Cost share of value-added and energy in the KLEM aggregate in sector *i* and region *r* ($i \notin FF$)
θ_{ir}^{KL}	Cost share of value-added in the KLE aggregate in sector *i* and region *r* ($i \notin FF$)
θ_{jir}^{NE}	Cost share of non-energy input *j* in the non-energy aggregate in sector *i* and region *r* ($i \notin FF$)
θ_{ir}^{Q}	Cost share of natural resources in sector *i* and region *r* ($i \notin FF$)
θ_{Tir}^{FF}	Cost share of good *j* (*T*=*j*) or labor (*T*=*L*) or capital (*T*=*K*) in sector *i* and region *r* ($i \in FF$)
θ_{isr}^{M}	Cost share of imports of good *i* from region *s* to region *r*
θ_{ir}^{A}	Cost share of domestic variety in Armington good *i* of region *r*

Key: KLEM – value-added, energy and non-energy; KLE – value-added and energy

Table B.4: Elasticities

σ_{ir}^{KL}	Substitution between labor and capital in value-added composite
σ_{ir}^{ELE}	Substitution between electricity and the fuel composite
σ_{ir}^{CGO}	Substitution between coal, gas and refined oil in the fuel composite
σ_{ir}^{KLE}	Substitution between energy and value-added in production
σ_{ir}^{KLEM}	Substitution between material and the KLE composite in production
σ_{jir}^{NE}	Substitution between material inputs into material composite
σ_{ir}^{Q}	Substitution between natural resources and other inputs in fossil fuel production
σ_{ir}^{M}	Substitution between imports from different regions
σ_{ir}^{A}	Substitution between the import aggregate and the domestic input

Table B.5: Endowments and emissions coefficients

$\overline{L}_r$	Base-year aggregate labor endowment in region *r*
$\overline{K}_r$	Base-year aggregate capital endowment in region *r*
$\overline{Q}_{ir}$	Base-year endowment of natural resource *i* in region *r* ($i \in FF$)
$\overline{G}_r$	Base-year public good provision in region *r*
$\overline{I}_r$	Base-year investment demand in region *r*
$\overline{B}_r$	Base-year balance of payment deficit or surplus in region *r*
$\overline{CO2}_r$	CO_2 emission endowment for region *r*
$a_{jir}^{CO_2}$	CO_2 emissions coefficient for fuel *j* (coal, gas, refined oil) in sector *i* and region *r*

Zero-profit conditions

Production of goods except fossil fuels

Production of commodities other than primary fossil fuels ($i \notin FF$) is captured by four-level constant elasticity of substitution (CES) cost functions describing the price-dependent use of capital, labor, energy, and material in production. At the top level, a CES composite of intermediate material demands trades off with an aggregate of energy, capital, and labor subject to a CES. At the second level, a CES function describes the substitution possibilities between intermediate demand for the energy aggregate and a value-added composite of labor and capital. At the third level, a CES function captures capital and labor substitution possibilities within the value-added composite, and likewise the energy composite is a CES function of electricity and a fuel aggregate. At the fourth level, coal, gas, and (refined) oil enter the fuel aggregate at a CES.

The unit-profit function for the value-added composite is:

$$\Pi_{ir}^{KL} = p_{ir}^{KL} - \left[\theta_{ir}^{K} v_r^{1-\sigma_{ir}^{KL}} + \left(1-\theta_{ir}^{K}\right) w_r^{1-\sigma_{ir}^{KL}}\right]^{\frac{1}{1-\sigma_{ir}^{KL}}} \leq 0 \qquad (22)$$

The unit-profit function for the energy composite is:

$$\Pi_{ir}^{E} = p_{ir}^{E} - \left[\theta_{ir}^{ELE} {p_{ELE,r}^{A}}^{1-\sigma_{ir}^{ELE}} + \left(1-\theta_{ir}^{ELE}\right)\left(\sum_{j \in CGO} \theta_{jir}^{CGO}\left(p_{jr}^{A} + p_r^{CO_2} a_{jir}^{CO_2}\right)^{1-\sigma_{ir}^{CGO}}\right)^{\frac{1-\sigma_{ir}^{ELE}}{1-\sigma_{ir}^{CGO}}}\right]^{\frac{1}{1-\sigma_{ir}^{ELE}}} \leq 0 \qquad (23)$$

The value-added composite and the energy composite enter the unit-profit function at the top level together with a CES composite of non-energy (material) intermediate input:[25]

[25] Note that the specification of the unit-profit function also includes the production of final demand components for private consumption (i=C), public consumption (i=G), and composite investment (i=I). In these cases, entries in the value-added nest are zero.

$$\Pi_{ir}^{Y} = p_{ir}^{Y} - \left[\theta_{ir}^{KLE} \left[\theta_{ir}^{KL} p_{ir}^{KL^{1-\sigma_{ir}^{KLE}}} + \left(1-\theta_{ir}^{KL}\right) p_{ir}^{E^{1-\sigma_{ir}^{KLE}}} \right]^{\frac{1-\sigma_{ir}^{KLEM}}{1-\sigma_{ir}^{KLE}}} + \left(1-\theta_{ir}^{KLE}\right) \left(\sum_{j \notin NE} \theta_{jir}^{NE} p_{jr}^{A^{1-\sigma_{ir}^{NE}}} \right)^{\frac{1-\sigma_{ir}^{KLEM}}{1-\sigma_{ir}^{NE}}} \right]^{\frac{1}{1-\sigma_{ir}^{KLEM}}} \leq 0 \qquad (24)$$

Production of fossil fuels

In the production of primary fossil fuels ($i \in FF$) all inputs except for the sector-specific fossil-fuel resource are aggregated in fixed proportions. This aggregate trades off with the sector-specific fossil-fuel resource at a CES. The unit-profit function for primary fossil fuel production is:

$$\Pi_{ir}^{Y} = p_{ir}^{Y} - \left[\theta_{ir}^{Q} q_{ir}^{1-\sigma_{ir}^{Q}} + \left(1-\theta_{ir}^{Q}\right) \left(\theta_{Lir}^{FF} w_r + \theta_{Kir}^{FF} v_r + \sum_{j} \theta_{jir}^{FF} \left(p_{ir}^{A} + p_r^{CO_2} a_{jir}^{CO_2} \right) \right)^{1-\sigma_{ir}^{Q}} \right]^{\frac{1}{1-\sigma_{ir}^{Q}}} \leq 0 \qquad (25)$$

Imports aggregate across regions

Imports of the same variety from different regions enter the import composite subject to a CES. The unit-profit function for the import composite is:

$$\Pi_{ir}^{M} = p_{ir}^{M} - \left[\sum_{s} \theta_{isr}^{M} p_{is}^{Y^{1-\sigma_{ir}^{M}}} \right]^{\frac{1}{1-\sigma_{ir}^{M}}} \leq 0 \qquad (26)$$

Armington aggregate

All goods used on the domestic market in intermediate and final demand correspond to a (Armington) CES composite that combines the domestically produced good and a composite of imported goods of the same variety. The unit-profit function for the Armington aggregate is:

$$\Pi_{ir}^{A} = p_{ir}^{A} - \left[\theta_{ir}^{A} p_{ir}^{Y^{1-\sigma_{ir}^{A}}} + \left(1-\theta_{ir}^{A}\right) p_{ir}^{M^{1-\sigma_{ir}^{A}}} \right]^{\frac{1}{1-\sigma_{ir}^{A}}} \leq 0 \qquad (27)$$

Market-clearance conditions

Labor

Labor is in fixed supply. The market-clearance condition for labor is:

$$\bar{L}_r \geq \sum_i Y_{ir} \frac{\partial \Pi_{ir}^Y}{\partial w_r} \tag{28}$$

Capital

Capital is in fixed supply. The market-clearance condition for capital is:

$$\bar{K}_r \geq \sum_i Y_{ir} \frac{\partial \Pi_{ir}^Y}{\partial v_r} \tag{29}$$

Natural resources

Natural resources for the production of primary fossil fuels ($i \in FF$) are in fixed supply. The market-clearance condition for the natural resource is:

$$\bar{Q}_{ir} \geq Y_{ir} \frac{\partial \Pi_{ir}^Y}{\partial q_{ir}} \tag{30}$$

Energy composite

The market-clearance condition for the energy composite is:

$$E_{ir} \geq Y_{ir} \frac{\partial \Pi_{ir}^Y}{\partial p_{ir}^E} \tag{31}$$

Value-added composite

The market-clearance condition for the value-added composite is:

$$KL_{ir} \geq Y_{ir} \frac{\partial \Pi_{ir}^Y}{\partial p_{ir}^{KL}} \tag{32}$$

Output

Domestic output enters Armington demand and import demand by other regions. The market-clearance condition for domestic output is:

$$Y_{ir} \geq \sum_j A_{jr} \frac{\partial \Pi_{jr}^Y}{\partial p_{ir}^Y} + \sum_s M_{is} \frac{\partial \Pi_{js}^Y}{\partial p_{ir}^Y} \qquad (33)$$

Armington aggregate

Armington supply enters all intermediate and final demands. The market-clearance condition for domestic output is:

$$A_{ir} \geq \sum_j Y_{jr} \frac{\partial \Pi_{jr}^Y}{\partial p_{ir}^A} \qquad (34)$$

Import aggregate

Import supply enters Armington demand. The market-clearance condition for the import composite is:

$$M_{ir} \geq A_{ir} \frac{\partial \Pi_{ir}^A}{\partial p_{ir}^M} \qquad (35)$$

Public consumption

Production of the public good composite ($i=G$) covers fixed government demand. The market-clearance condition for the public good composite is:

$$Y_{Gr} \geq \overline{G}_r \qquad (36)$$

Investment

Production of the investment good composite ($i=I$) covers fixed investment demand. The market-clearance condition for composite investment is:

$$Y_{Ir} \geq \overline{I}_r \qquad (37)$$

Private consumption

Production of the composite private consumption good (i=C) covers private consumption demand. The market-clearance condition for composite private consumption is:

$$Y_{Cr} \geq \frac{INC_r}{p_{Cr}^{Y}} \tag{38}$$

Carbon emissions

A fixed supply of CO_2 emissions limits demand for CO_2 emissions. The market-clearance condition for CO_2 emissions is:

$$\overline{CO2}_r \geq \sum_j \sum_i Y_{ir} \frac{\partial \Pi_{ir}^{Y}}{\partial (p_{jr}^{A} + a_{jir}^{CO_2} p_r^{CO_2})} a_{jir}^{CO_2} \tag{39}$$

Income-balance conditions

Income balance

Net income of the representative agent consists of factor income and revenues from CO_2 emission regulation adjusted for expenditure to finance fixed government and investment demand and the base-year balance of payment. The income-balance condition for the representative agent is:

$$INC_r = w_r \overline{L}_r + v_r \overline{K}_r + \sum_{i \in FF} q_{ir} \overline{Q}_{ir} + p_r^{CO_2} \overline{CO2}_r - p_{Ir}^{Y} \overline{Y}_{Ir} - p_{Gr}^{Y} \overline{Y}_{Gr} + \overline{B}_r \tag{40}$$

Appendix C: Generation of pseudo random Armington elasticities for Monte Carlo simulations

We first generate $i \in I = \{1, 2, ..., n\}$ sector specific gamma distributed variables, $\gamma_{s,i}$, and one common economy wide gamma distributed variable, γ_c. Let u_i be $i \in I$ draws from the standard uniform probability distribution. These pseudo-random numbers are generated using GAMS (numerical software). We transform the u_i 's to draws from the two-parameter gamma distribution, $G(\alpha_s, \beta_s)$, by solving:

$$\frac{1}{\Gamma(\beta_s)} \int_0^{\gamma_{s,i}/\alpha_s} t^{\beta_s - 1} e^{-\tau} d\tau = u_i, \quad \forall i \in I, \tag{41}$$

for $\gamma_{s,i}$; i.e., $\gamma_{s,i}$ is a random draw from $G[\alpha_s, \beta_s]$. Here, the denominator $\Gamma(\beta_s) = \int_0^\infty \tau^{\beta_s - 1} e^{-\tau} d\tau$ is the gamma function.[26] Note that the gamma variables have expectations $E[G(\alpha_s, \beta_s)] = \alpha_s \beta_s$ and variances $\text{var}[G(\alpha_s, \beta_s)] = \alpha_s^2 \beta_s \equiv \sigma_s^2$. We generate draws from a second gamma distribution $G(\alpha_c, \beta_c)$, γ_c, by the same procedure.

Let the stochastic Armington elasticity of sector $i \in I$ be given by:

$$A_i = \bar{A}_i + \theta(\gamma_c - \alpha_c \beta_c) + (1 - \theta)(\gamma_{s,i} - \alpha_s \beta_s). \tag{42}$$

Here, $\theta \in [0,1]$ is a constant and $\bar{A}_i$ is the benchmark Armington elasticity for sector $i \in I$. Note that the realized Armington elasticity, A_i, is the constant $\bar{A}_i$ plus a linear combination of a 'shock' that hits all sectors, $\gamma_c - \alpha_c \beta_c$, and a sector specific shock $\gamma_{s,i} - \alpha_s \beta_s$.

Equations (41) and (42) implicitly define a random variable A_i for the Armington elasticity of sector $i \in I$, with expectation $E[A_i] = \bar{A}_i$. Further, the variance is given by $\text{var}[A_i] = \theta^2 \sigma_c^2 + (1-\theta)^2 \sigma_s^2 + 2\theta(1-\theta)\text{cov}[\gamma_c, \gamma_{s,i}] = \theta^2 \sigma_c^2 + (1-\theta)^2 \sigma_s^2$, whereas the covariance between the Armington elasticities of two sectors *i* and *j* is $\text{cov}[A_i, A_j] = E[A_i A_j] - E[A_j]E[A_i] = E[\theta^2 \gamma_c^2 + \bar{A}_i \bar{A}_j] - \bar{A}_i \bar{A}_j = \theta^2 \sigma_c^2$ ($j \in I \setminus \{i\}$). It follows that the correlation coefficient can be expressed as:

[26] We restrict $1.0 \times 10^{-8} \le u_i \le 1 - 1.0 \times 10^{-8}$ to avoid $u_i = 0$ or $u_i = 1$ when solving equation (41).

$$corr\left(A_i, A_j\right) = \frac{\theta^2 \sigma_c^2}{\theta^2 \sigma_c^2 + \left(\theta - 1\right)^2 \sigma_s^2}, \tag{43}$$

which satisfies $corr\left(A_i, A_j\right) = 1$ if $\theta = 1$ and $corr\left(A_i, A_j\right) = 0$ if $\theta = 0$.

We want to generate a probability distribution for the Armington elasticities with a specific correlation coefficient $corr\left(A_i, A_j\right) = \rho$ for use in the numerical simulations. Then, for any given triple $\left(\rho, \sigma_c^2, \sigma_s^2\right)$, equation (43) implies that the constant θ must solve:

$$\theta = \frac{\rho\sigma_s^2 - \sigma_c^2 \sqrt{\rho\left(1-\rho\right)\sigma_s^2 / \sigma_c^2}}{\rho\left(\sigma_s^2 + \sigma_c^2\right) - \sigma_c^2}, \tag{44}$$

with $\theta = 1/2$ if $\rho\left(\sigma_s^2 + \sigma_c^2\right) = \sigma_c^2$.

A histogram of the Armington elasticities generated using equations (41) and (42) with scale parameters $\alpha_c = \alpha_s = 5/4$, shape parameters $\beta_c = \beta_s = 3$, correlation $corr\left(A_i, A_j\right) = 0$ and expectation $\overline{A}_i = 4$ is given in Figure A1. A 95% prediction interval for A_i is given by $[1.0, 9.3]$ with these parameters. The median of the sample with $n = 5000$ simulation runs graphed in Figure A1 is 3.6. Equations (41) and (42) do not guarantee a non-negative Armington elasticity. This is not a problem given the selected parameter values (all draws turn out to be positive; see figure C1).

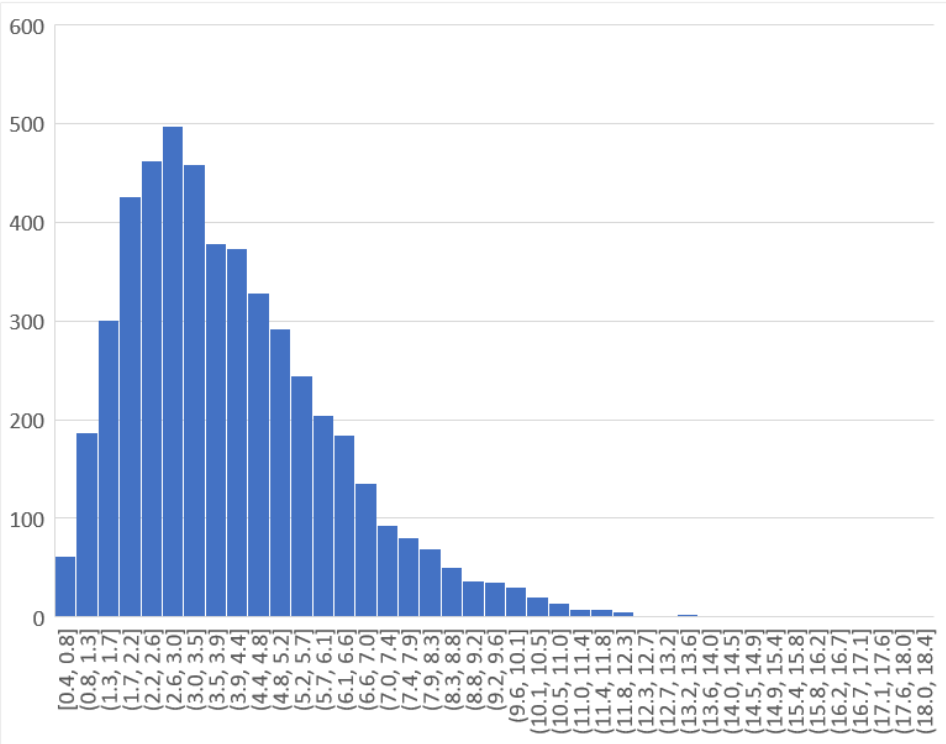

Figure C1. Example of generated Armington elasticities. Histogram. Sample size is 5000.