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Abstract

This paper studies the importance of demand and supply shocks in the oil market, and tries to explain the formation of the short-run oil price by applying an extended commodity storage model to the cyclical components of the price. First, I employ a multivariate method to extract the cyclical component of the oil price, world oil consumption, and global GDP. Next, I find a large and positive effect of global GDP shock on the oil price cycles in a VAR model. Then, I estimate the commodity storage model using a moment-matching method. All parameters are estimated significantly, and the model shows good capability of reproducing the volatility and persistence of oil price cycles. I find that the GDP shock generates a much more moderate effect on the oil price cycles in the extended commodity storage model than the empirical evidence from the VAR analysis, and the production shock plays an important role for the variance of the cyclical component of the oil price.

JEL Classification: *C15, G1, O13, Q4.*

Keywords: *Oil price, demand shock, supply shock, competitive storage model, Beverage-Nelson decomposition, simulated method of moments.*

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1 Introduction

Crude oil is one of the most frequently traded commodities in the global commodity market. The real oil price features high volatilities and occasionally large spikes. For example, the real WTI price of crude oil increased by more than 400% from 1999 to 2008. What are the main driving forces behind the dynamic behavior of the oil price? Are changes in supply or demand behind behind these large fluctuations? Scholars have studied factors affecting the oil prices from different angles. On one hand, Griffin (1985), Pindyck (1994), Alhajji and Huettner (2000), Hansen and Lindholt (2008), Lawell (2013) and others have proposed theories where OPEC controls the supply of oil. On the other hand, Cooper (2003), Krichene (2006), Hamilton (2009), and many others have proposed demand-driven explanations.

There is, however, a need to use dynamic structural models to evaluate the relative importance of demand and supply factors. The first step is then to build a theoretical model for the oil market and estimate the structural shocks. The aim of this paper is to study what element explains the formation of oil prices in a structural model, and to quantify the relative importance of demand shocks and supply shocks to the variance of oil price.

To this end, I put forward an extended commodity storage model to explain cyclical components of the oil price, taking into account exogenous variation in both global income and crude oil supply. I use a simulated method of moments (SMM) to estimate the structural parameters of the model using data from the period 1986–2009. I show that the extended storage model represents a major improvement over the original traditional commodity storage model in Deaton and Laroque (1992; 1996). I find that GDP shocks generate very moderate effects on the oil price, while production shocks explain a large fraction of the oil price variance.

In this paper, I first extract the cyclical component of quarterly data on oil price, global oil quantity, and world GDP by using a multivariate Beveridge-Nelson (BN) decomposition method for the period from 1986, after the substantial price decrease in the oil market, to the end of the financial crisis in 2009. In the BN decomposition, I show that there is a positive contemporaneous correlation between the cyclical component of global oil quantity and world gross domestic product (GDP). I also document a large and positive effect of world GDP shock on the oil price cycles in a VAR model.

Second, to understand the dynamics of crude oil price cycles, I employ a competitive storage model that is devised with both stochastic income and production process without serial correlation; this is an extended version from the model in Deaton and Laroque (1992, 1996), where only production shock was considered. In this model, the risk-neutral and profit-maximizing speculator or institutional investor holds inventories from one period to the next when expected future prices are equal to the current price. The speculative storage has a smoothing effect on the equilibrium price by reducing the volatility and introducing persistence.

Third, to gain insight into the role of income shock and production shock, I investigate two simplified commodity storage models with single stochastic production or income process. I solve these storage models by using collocation methods that assume the same shock volatility. I find that the production shock introduces a more intensive smoothing effect on the equilibrium price with a lower variance and higher persistence than the income shock.

Subsequently, I apply the extended storage model to the cyclical components of oil price, world oil consumption, and global GDP through using a SMM. I also compare the estimates and simulated moments between the extended storage model and the storage model with only production shocks as in Deaton and Laroque (1992). The extended storage model shows a better fitness to the data. All the coefficients of the extended storage model are estimated to be significant. The estimated short-run demand elasticity is –0.2, which is close to the results garnered by Gately and Huntington (2002), Cooper (2003), and Dées et al. (2007). By employing the estimated parameters, the extended storage model (with both stochastic production and income processes) reproduces some important features of the oil price cycles. In particular, it generates a high price volatility and persistence that are similar to real oil price cycles.[1] On the other hand, one of the estimates from the storage model with only production shocks is statistically insignificant, and the model is not able to match many data moments.

Finally, I perform a counterfactual analysis and compute impulse responses of oil prices to exogenous income and production shocks. I find that the model generates moderate results for the effect of income shocks on oil price cycles. The supply variation explains the largest fraction of the variance in the oil price cycles due to the large estimated production shock volatility.

Increasingly, scholars have been trying to explain the long-run determinants of crude oil prices. The topic in this paper, however, relates more to short-run analyses. For example, Cooper (2003) employed a multiple linear regression analysis and estimated short-run elasticities of crude oil demand in both OECD and non-OECD countries by using ordinary least squares (OLS), over the period 1979–2000. He found that crude oil demand displays high price inelasticity in the short run. Furthermore, Krichene (2006) has also examined the world oil market in a single equation estimation and has concluded that there is less elastic demand but a high demand income elasticity for crude oil in short run. Contrary to previous schol-

[1]The extended storage model generates a higher autocorrelation than the canonical model in Deaton and Laroque (1992, 1996).

arship, this paper studies the driving force of the short-run oil price in a nonlinear structural model for the recent period of 1986–2009. Rather than the single estimation method, the structural parameters are estimated using the SMM method.

This paper is relevant to the literature that studies the determinant of oil price in VAR models. The seminal paper of Kilian (2009) has discussed the effects of demand shocks and supply shocks in a three-variable structural vector autoregressive (VAR) model. Kilian (2009) employed the Cholesky decomposition method to identify shocks; it is assumed supply is vertical and does not respond to demand shocks and price shocks simultaneously. Later, Kilian and Murphy (2014) extended the work by Kilian (2009) with speculation using data on oil price, production, global activity, and inventory. The authors have shown that demand shocks are still the main cause of fluctuations in the price of oil for the period 2003–2008. They did not find any strong evidence for speculative shocks during the oil price surge in 2003–2008. However, they found that speculation played an important role in an earlier period, 1986–1990. Juvenal and Petrella (2015), in revising and extending the work of Kilian and Murphy (2014), have assessed the role of both speculative oil demand and supply shocks in a factor-augmented vector autoregressive (FAVAR) model. They have found that speculation has had significant effects on the increase of oil prices since 2004. In addition, the most recent paper by Baumeister and Hamilton (2018) has discussed the relative importance of oil supply and demand shocks using a VAR model incorporating uncertainties to identify shocks using Bayesian method. They have found that the oil supply shock plays an important role in the variation of the oil price. Similar results are also found in Caldara et al. (2016) using an identified structural VAR model.

The empirical model presented in this paper is also connected to scholars that advocates the use of commodity storage models to derive the implications for oil and other commodity prices. The canonical commodity storage model was first developed by Williams and Wright (1991). Deaton and Laroque (1992, 1996) made the first attempts to confront the theoretical model with actual commodity prices. They used annual observations for thirteen commodities, including copper, palm oil, and other agricultural goods, for the period 1900–1987. The model in Deaton and Laroque (1992, 1996) is able to match the volatility, skewness, and kurtosis of commodity prices. However, the model cannot explain the high persistence that has been observed in commodity prices. In their model, the storage acts by "leaning against the wind", which stabilizes the production shock effect on the commodity prices between consecutive periods. Dvir and Rogoff (2010) have augmented the model of Deaton and Laroque (1992, 1996) by introducing stochastic growth dynamics into the income process. The storage, in fact, amplifies the income shock on prices. Nevertheless, Dvir and Rogoff (2010) did not apply their theoretical model to fit the empirical data on oil prices.

This paper also employs an augmented storage model for examining the effect of storage on the oil market. In contrast to previous papers, I assume that both production and income processes are stochastic. On account of such an assumption, I am able to evaluate the relative importance of supply and demand shocks to variations in the oil price. Moreover, this paper also validates the theoretical model by applying it to the empirical data pertaining to the oil market for the period 1986–2009.

The remainder of the paper is divided in five sections. In section 2, I first introduce the data sources. Then I explain the multivariate BN decomposition method and discuss some stylized facts of oil cycles based on decomposed data. In section 3, I discuss the effect of a GDP shock on the price of oil in a VAR model using cyclical components of oil series. After that, in section 4, I describe my extended storage model incorporating stochastic production and income shock. I also explain the role of shocks in the simplified models. In section 5, I apply the extended storage model to the cyclical components of the oil price, world oil consumption and global GDP by using the SMM method. On the basis of the counterfactual analysis, I discuss the importance of income and production shocks. Employing estimated parameters, I also compute the impulse response of the oil price to a GDP shock in the extended storage model and compare it to the empirical evidence from the VAR model. Finally, in section 6, I provide some concluding remarks.

2 Oil market cycles

In this section I provide some facts about the oil market cycles through using empirical data. First, I introduce the data sources that have been employed for this analysis. Second, I explain the multivariate decomposition method for extracting cyclical components within the price of oil, word oil consumption, and global GDP.

2.1 Data sources

In this paper, quarterly data spanning the period 1986:Q1–2009:Q4 are used to estimate the model and assess its validity. The price of crude oil is the West Texas Intermediate (WTI) price[2], as taken from EIA (2016). Data on oil production and the total stock of crude oil in OECD countries were obtained from the EIA (2016). World production of crude oil minus the change in the OECD stock of crude oil is used as a measure for total consumption of

[2]The quarterly observation of Brent oil price does not have large difference from the WTI price for the sample period. It has similar volatility and persistence as the WTI price. The standard deviation of the nominal WTI price is 20.35, and its autocorrelation is 0.96. The Brent price of crude oil has a standard deviation as of 20.37, and a autocorrelation as of 0.97.

(demand for) crude oil. The GDP is computed by using the world GDP index, which is collected from Fagan et al. (2001) and transformed into GDP levels by using the annual GDPs from the World Bank (2014).[3] The nominal price of oil and GDP is deflated by using the U.S. CPI, taken from U.S. Bureau of Labor Statistics (2014).[4]

2.2 Extracting the cyclical component

Many studies have confirmed the comovement among oil price, global oil consumption, and world GDP (Hansen and Lindholt, 2008; Golombek et al., 2018). Cointegration tests have suggested that there is a long-run equilibrium among variables for different sample periods and from diverse data sources. Thus, the standard univariate detrending methods, such as the Hodrick-Prescott (HP) filter and the band pass filter, may not be proper and may suffer from misspecification. Furthermore, Harvey and Jaeger (1993), Cogley and Nason (1995) and Canova and Ferroni (2011) have criticized the fact that the Hodoric-Prescott (HP) filter fails to remove the stochastic trend and produces "spurious cycle" phenomena.[5] Cochrane (1988) discusses BN decomposition, introduced by Beveridge and Nelson (1981), gives a sensible definition of the trend component. The trend is the sum of the current variable and all the future expected changes. Therefore, in this paper, I employ the multivariate BN decomposition method, as the data transformation method for oil series. Similar method is also used in Cochrane (1994).

2.2.1 The Beveridge-Nelson decomposition method

The BN decomposition method is a model-based method for isolating series into permanent trend components and cyclical transitory components. Let us first assume a cointegrated vector $\mathbf{X}_t$. Then the Wold representation of $\triangle\mathbf{X}_t$ takes the form:

$$\triangle\mathbf{X}_t = \delta + \mathbf{\Psi}\left(L\right)\epsilon_t, \tag{1}$$

where $\mathbf{\Psi}\left(L\right) = \sum_{k=0}^{\infty}\psi_k L^k$ and $\psi_0 = 1$. δ is the deterministic trend growth rate and $\delta = E(\Delta\mathbf{X})$, $\mathbf{\Psi}$ denotes the coefficient vector, and ϵ_t is the residual in the Wold representation

[3] I first compute a factor as the ratio between the annual GDP level in 2009 and the average GDP index using quarterly observations in that year. The quarterly GDP level is computed as the product of the quarterly index and the constant factor.

[4] The U.S. CPI is commonly used as an measure of world inflation on average. Almoguera et al. (2011), Lin (2011) and Kilian and Murphy (2014) also use the U.S. CPI as the deflator of world GDP and oil prices.

[5] Their discussions are mainly focused on real business cycle models.

of $\triangle \mathbf{X}_t$. Then the trend component follows a unit root process with a drift, such as

$$\mathbf{X}_t^T = \delta + \mathbf{X}_{t-1}^T + \mathbf{\Psi}(1)\,\epsilon_t.$$

Using the recursive substitution for the Wold representation in (1) we can write X_t as a function of all the shocks, such that

$$\mathbf{X}_t = \mathbf{X}_0 + \delta t + \mathbf{\Psi}(1)\sum_{s=1}^{t}\epsilon_s + (1-L)\,\tilde{\Psi}(L)\sum_{j=1}^{t}\epsilon_t, \tag{2}$$

where $\tilde{\Psi}(L) = \sum_{j=0}^{\infty}\tilde{\Psi}_j L^j$ and $\tilde{\Psi}_j = -\sum_{k=j+1}^{\infty}\psi_k$.

By following the example of Beveridge and Nelson (1981), the trend component of a vector $\mathbf{X}_t$ is defined as the limiting forecast as horizon goes to infinity, adjusted for the mean growth rate

$$\mathbf{X}_t^T \equiv \lim_{h\to\infty}\mathbf{X}_{t+h|t} - \delta h = \mathbf{X}_t + \sum_{i=1}^{\infty}\left(E_t\triangle\mathbf{X}_{t+i} - \delta\right). \tag{3}$$

Equation (3) also implies that if the variable $\mathbf{X}_t$ is forecasted to rise, its level is below the trend. Inserting (2) into (3), I obtain the trend component as follow,

$$\mathbf{X}_t^T = \mathbf{X}_0 + \delta t + \mathbf{\Psi}(1)\sum_{s=1}^{t}\epsilon_s. \tag{4}$$

In this equation, $\mathbf{X}_0$ is the initial value of $\mathbf{X}_t$ in period zero, $\mathbf{\Psi}(1)$ measures the long-run impact of forecast error, and δt represents deterministic trend. Furthermore, the cyclical component at time t can be computed by employing the following equation:

$$\mathbf{X}_t^C = \mathbf{X}_t - \mathbf{X}_t^T = (1-L)\,\tilde{\Psi}(L)\sum_{j=1}^{t}\epsilon_t. \tag{5}$$

The term $(1-L)\,\tilde{\Psi}(L)$ is the measure of transitory impact of forecast errors.

According to equation (4) and (5), the implementation of BN decomposition on the crude oil price, world oil consumption and global GDP indicates that oil market growth consists of both deterministic and stochastic trends. The cyclical components of oil occur due to the fluctuations in the structural growth of the oil market.

Table 1: Cointegration Test

(a) ADF Test of Unit Root					
	No.lag	Test statistic	5% Critical value	p-value	Conclusion
World consumption: lnQ	1	-4.798	-3.459	0.001	I(0)
World GDP: lnY	0	0.777	-3.458	0.999	I(1)
Price of oil: lnP	0	-2.240	-3.458	0.472	I(1)
(b) Johansen Test for the Existence of Cointegration Vectors					
Cointegrating rank	0	1	2		
Trace statistics	68.592	30.142	5.854		
5% critical value	35.193	20.262	9.164		
p-value	0.001	0.002	0.204		
Number of obs.	96				
Differenced lags	1				

Notes: Table (a) shows the ADF statistics with a drift and deterministic trend for the unit root process of each variable. The number of lags used is selected by the AIC and BIC index. The critical value of rejecting the null hypothesis at 5 percent level and the p-value of the statistics are presented. The conclusion of the ADF test at 5 percent significance level is listed. Table (b) shows the Johansen test of vector cointegration for different cointegration ranks. This vector includes $\ln Q$, $\ln Y$, and $\ln P$. is the coefficient for the lagged vector in the VEC model. 96 observations are used in the tests.

2.2.2 Implementation of multivariate BN decomposition for oil data

To evaluate the applicability of multivariate BN decomposition for the oil price, world oil consumption, and global GDP, I employ a state-space approach (see Cochrane (1994) and Morley (2002)). I perform the decomposition in two steps. First, I perform a Johansen's unrestricted cointegration rank test (trace test) among logarithm values of oil consumption, world GDP, and oil price based on a VEC model of vector $\mathbf{X}_t = [\ln GDP_t, \ln OilQuantity, \ln Price_t]$ such that

$$\triangle \mathbf{X}_t = \beta + \Pi \mathbf{X}_{t-1} + A_1 \triangle \mathbf{X}_{t-1} + u_t, \tag{6}$$

where u_t denotes the residual. The augmented Dickey–Fuller test (ADF) tests as shown in Table 1 indicates that world GDP and the crude oil price are I(1). However, world oil quantity is a stationary series. Furthermore, the cointegration test results in Table 1 show that both the null hypothesis of rank(Π)$\leq$0 and rank(Π)$\leq$1 are rejected at 5 percent significant level. But I can not reject the null hypothesis of rank(Π)$\leq$2. It suggests that there are at least one cointegrating relationship within the system. Second, I decompose trend and cyclical components by using estimates from the VEC model in equation (6), with detected numbers

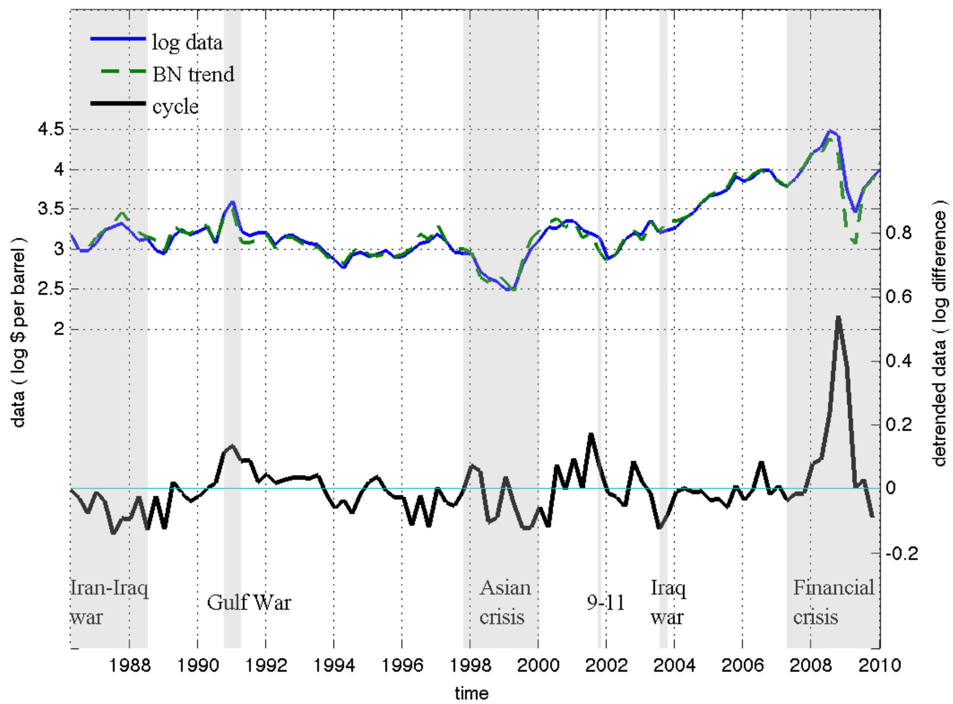

Figure 1: Crude oil prices: $\ln P$

Notes: The figure plots the real crude oil price in logarithm value, the BN trend, and the cyclical component of the real oil price. The real price of oil is measure in 1996 USD. The left scale is for the price data. The right scale is for the deviations from the trend and has units as percent deviations from the trend. The shaded area indicates specific period when important events were taking place in the oil market. These events include the Iran-Iraq war (1986:Q1–1988:Q2), the Gulf War (1990:Q3–1991:Q1), the Asian financial crisis and oil crisis (1997:Q2–1999:Q4), the 9/11 attacks (2001:Q3), the invasion of Iraq (2003:Q2–2003:Q3), and the global financial crisis (2008:Q1–2009: Q4).

of cointegration relationship.[6] The approach in details is summarized in Appendix A.

Properties of the transformed data

After steps 1 and 2, I decompose the global GDP, oil consumption, and oil price into trend and cyclical components. Figure 1 plots the trend and cyclical components of the real oil price. The bold solid line in Figure 1 denotes the cyclical component in price. It is measured in terms of its percentage deviation from the trend component. The cyclical component varies around the zero-mean level.

The correlation between the trend and cyclical components of oil price is negative, –0.61. Thus an increase in the price trend is associated with a decrease in the cycle component,

[6]The VEC in equation (6) can be rewritten into an ARIMA(2,1,0) process, such that

$$\mathbf{X}_t = \beta + \mathbf{X}_{t-1} + (\pi + A_1)\,\mathbf{X}_{t-1} - A_1\mathbf{X}_{t-2} + u_t.$$

while, according to Stock and Watson (1988), the variations in the cycle component indicate adjustments towards the shifting trend. This finding suggests that the cycle component will decrease initially with an increase in the price trend and that this results in a lower net increase in the oil price. The negative impact of cyclical innovations is temporary and dissipates over time.

Furthermore, Figure 1 shows that the fluctuations in the cycle component are consistent with the anecdotal evidence on the relative importance and timing of the fluctuations in the global crude oil market, which helps to verify the proposed decomposition. For instance, the price cycle became negative after Saudi Arabia changed its policy from reducing supply to maintain the price to increasing supply to maintain market share from 1986. Then the oil price fell substantially, while the price cycle increased sharply at the beginning of 2007 and dropped dramatically due to the financial crisis in late 2008.

The relationship between cyclical components in the world GDP and the global oil quantity is discussed next. Figure 2 exhibits the evolution of cyclical components in the global oil consumption as measured against world GDP. The global oil consumption presents a pro-cyclical pattern—in other words, the global oil consumption has a positive contemporaneous correlation with world GDP. The correlation is 0.30 for the period 1986:Q1–2009:Q4 and 0.5 for the last part of this period, 1999:Q1–2009:Q1. It is apparent that during the recession period from late 2008 to 2009, there is a tight correlation between the cyclical components of global oil consumption and world GDP.

Furthermore, Table 2 shows the volatility, persistence, and correlation for cyclical components of the variables. Columns 1 and 2 show that the oil price cycle is 9.8 times as volatile as the world GDP cycle, where the cyclical component of world GDP is a 1 percent deviation from its trend, and the price cycle is a 9.8 percent deviation from the trend of the crude oil price. Furthermore, the world oil consumption cycle is more volatile than the world GDP cycle, where the consumption volatility is at 2.4 percent. In addition, column 3 shows the persistence of the series. The first order autocorrelation is 0.58 for the price cycle and 0.40 for the oil consumption cycle. Moreover, the persistence of the world GDP cycle is 0.62. The autocorrelation of world GDP is lower than 1, which confirms the stationary feature of the cyclical components of the world GDP.

3 VAR analysis

In this section, I discuss the features of the oil cycles by using a VAR model. The purpose of this analysis is to investigate the effect of GDP shock to the oil price cycles using the VAR

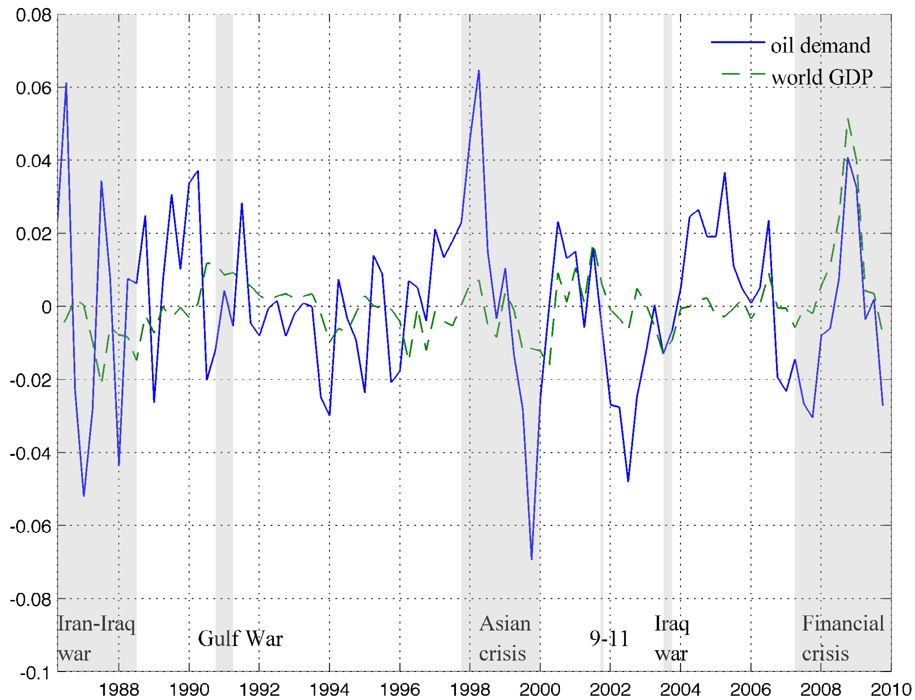

Figure 2: Comovement of cyclical components in the oil consumption and world GDP

Notes: The figure plots the cyclical components of global oil consumption and world GDP. The cyclical components have been estimated through using a multivariate BN decomposition method.

Table 2: Cyclical properties of the global oil market

Log values	Standard deviation	Relative standard deviation	1st-order autocorrelation
World GDP cycle	0.010	1.000	0.624
Oil price cycle	0.098	9.799	0.577
Oil consumption cycle	0.024	2.365	0.404

Notes: This table is generated by using the logarithm value of cyclical components.

model as in the seminal papers of recent literature in this branch (Kilian, 2009, Baumeister and Peersman, 2013, and Kilian and Murphy, 2014).

Let us consider a structural VAR model of the oil market for $z_t = [\ln Y_t, \ln Q_t, \ln P_t]$. In this formula, $\ln Y_t$, $\ln Q_t$ and $\ln P_t$ denote the logarithm values of cycle components in world GDP, global oil quantity, and oil price (see Figure 1 and 2). In following the design of Kilian (2009), the VAR model takes the following form:

$$z_t = \alpha + \sum_{i=1}^{K} A_i z_{t-i} + e_t, \tag{7}$$

where e denotes the reduced form residuals, A_i is the coefficient matrix of the autoregressive terms, and α is the constant vector. I impose a Cholesky assumption of long-run restrictions in order to identify GDP, supply, and price shocks. I denote ϵ as the structural residuals where $E\epsilon_t\epsilon_t' = I$. Then the Cholesky decomposition states that

$$e_t = \begin{pmatrix} e_t^Y \\ e_t^Q \\ e_t^P \end{pmatrix} = A\epsilon_t = \begin{bmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{bmatrix} \begin{pmatrix} \epsilon_t^Y \\ \epsilon_t^Q \\ \epsilon_t^P \end{pmatrix}, \tag{8}$$

and $AA' = Ee_te_t'$.

According to the restriction on A_0^{-1} in (8), I assume an exogenous process for the GDP cycle, where the GDP cycle is not affected by the supply shock ϵ_t^Q and the price shock ϵ_t^P in the same period. I also assume that the current production of oil has no response to instantaneous changes to the oil price.[7]

I estimate the structural VAR model in (7) by using the method of least squares. The estimated values are then employed to construct the impulse response results. Figure 3 plots the response of the oil price cycle to one standard deviation structural shocks on GDP, supply, and the price of oil. The structural shocks are assumed to be orthogonal. The shaded area in Figure 3 denotes the inference generated by using a bootstrap method for 5,000 replications of the simulation.

Figure 3 shows how a GDP shock causes a sharp and significant increase in the oil price. Furthermore, this positive response to the GDP shock lasts at least three quarters. This graphic depiction reveals that GDP is an essential factor that affects the oil price cycles, and this result, moreover, is consistent with that of Golombek et al. (2018), who have concluded that global income is the main driving force behind the oil price.

[7]These assumptions are different from those of Kilian (2009), who assumed a vertical short-run supply curve, while the shift in the aggregate supply is the result of the simultaneous change in the oil supply.

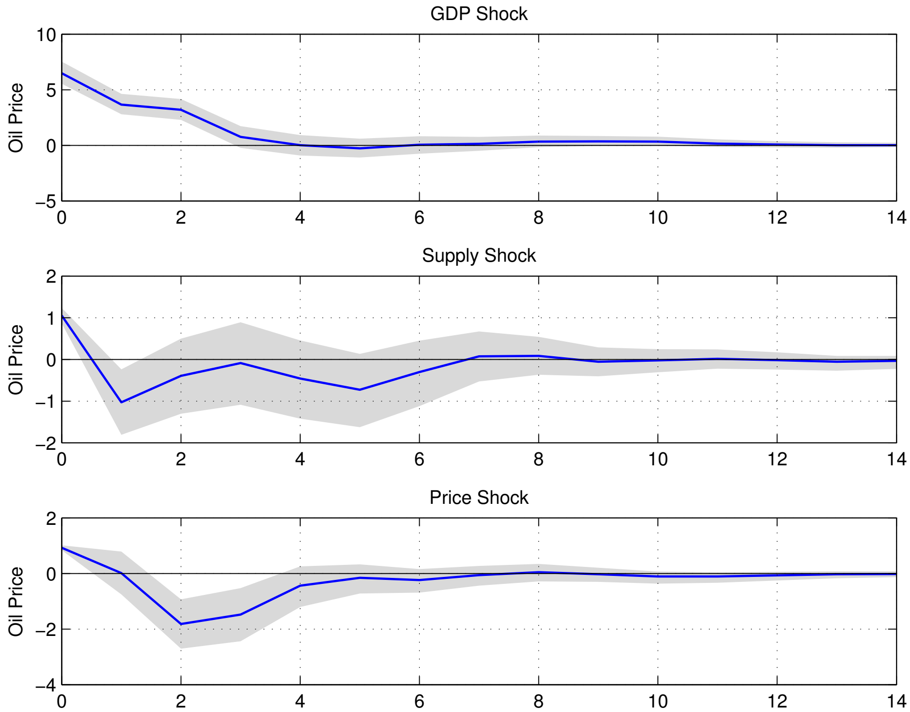

Figure 3: Impulse responses of the oil price cycle to structural innovations from a VAR model

Notes: The figure plots the responses of the oil price cycle to structural shocks of one standard deviation. The inference shown as the shaded area in the figure is constructed through using a bootstrap method for 5,000 replications.

The supply shock has a positive but lower effect on the oil price cycle when the positive supply shock occurs instantaneously; however, the response of the oil price cycle becomes negative in the ensuing periods as an effect of increasing oil supply, and then converges to zero over the time of simulation. One possible reason for the movement of price back to zero is that the production of oil increases in some countries (OPEC countries, for instance), and the other countries have delayed responses and react to the lower prices by reducing their supply, which in turn contributes to the adjustment back to zero of the oil price in the ensuing period.

The price shock may refer to unanticipated price changes that affect the expectations of oil availability in the market.[8] Figure 3 shows an ambiguous effect of price shock. The price shock has initial positive effect on the oil price cycle by construction, but the effect changes direction and becomes statistically insignificant after four quarters.[9]

To summarize, employing data on cyclical components of world GDP, oil quantity and

[8]Kilian (2009) has discussed how exogenous political events can be seen as an example of the price shock.

[9]Due to the ambiguous nature of the evidence, I am not focusing on the effect of the price shock in this paper.

crude oil price in a VAR model, I find a large and positive effect of world GDP shock on the oil price cycles. In the next two sections, I will discuss the relative importance of GDP shock in an extended storage model, as a comparison to the empirical evidence in the VAR model.

4 An extended commodity storage model

Considering the importance of world GDP shock (discussed in section 3), I introduce an additional shock into the model of Deaton and Laroque (1992, 1996) to capture fluctuations in the income process, while I retain the shock on production. In the following section, I start by describing the extended commodity storage model, including both stochastic income and production processes. Afterwards, I discuss the role of income and production shocks to the oil prices in simplified versions of the model, that is two single-shock models. I solve these two simplified models with fixed parameters and compare the moments of simulated data.

4.1 A storage model with income shocks and production shock

This analysis assumes a risk-neutral speculator who chooses to store crude oil by maximizing his/her aggregate net present value of profit within a discrete time framework. This speculator can be, for instance, seen as the OPEC-core countries who decide the extraction of crude oil or the amount of oil left under ground.[10] Whereas, the rest of the world does not have market power and their supply of oil is assumed to be exogenous in the short run for simplicity.

Availability

I consider A_t as the oil availability, also defined as "amount at hand" in Deaton and Laroque (1992, 1996), which measures the amount of oil available to be consumed in period t. This amount of oil has been produced at time t or an earlier period, and it has not been sold before the current period t . Accordingly, the availability of oil at period t is the sum of any storage carried from the previous period X_{t-1} and current production Z_t, such that the following equation can be formulated:

$$A_t = X_{t-1} + Z_t, \tag{9}$$

where $X \geq 0$. An alternative interpretation of X could be OPEC core's spare capacity. In other words, X can be seen as the measure of the extra production the OPEC core can produce, if the OPEC core group decides to increase supply in the short run. Following the

[10] Alhajji and Huettner (2000) and Golombek et al. (2018) find that OPEC or OPEC core exerts market power.

example of Dvir and Rogoff (2010), I assume a zero depreciation rate on carrying storage from previous periods for the sake of simplicity.[11]

At each point of time, the availability of oil, including the storage in earlier period, should equal the current consumption together with the inventory stored for the next period, such that the following equation can be formulated:

$$A_t = Q_t + X_t. \tag{10}$$

Demand function

Following the example of Dvir and Rogoff (2010), I assume an iso-elastic demand function of oil such that the oil price cycle is a function of oil demand and income,

$$P_t = \left(\frac{Q_t}{Y_t}\right)^{-\gamma}, \tag{11}$$

where $\gamma > 1$. According to equation (11) the price P_t is a Constant Relative Risk Aversion (CRRA) function that provides the effective demand of oil at rate γ. Accordingly, $\frac{1}{\gamma}$ is the price elasticity of demand. In this case, I implicitly assume a unit income elasticity of demand where the percent change in demand is equal to the percent change in income ($\partial \ln Q/\partial \ln Y = 1$). This assumption is in line with empirical results from several recent studies. For example, Gately and Huntington (2002) have found that income elasticities are around 1 for non-OECD countries; more recently, Golombek et al. (2018) have used a structural model to estimate an income elasticity of demand of 1.11.

Inserting equation (10) into (11), the inverse demand function can be written in terms of availability and income:

$$P_t = \left(\frac{A_t - X_t}{Y_t}\right)^{-\gamma}. \tag{12}$$

Income and production shocks

The production of oil is sensitive to the events in the producing area, which would suggest oil production amount fluctuates over time. Similar to the work of Deaton and Laroque (1992, 1996)[12], I assume that the production cycle Z_t fluctuates around a constant level $\bar{Z}$, such that

$$Z_t = \bar{Z} \exp\left(e_t^z\right), \tag{13}$$

where e_t^z is a stochastic production shock that follows a normal distribution $e_t^z \sim N\left(0, \sigma_z^2\right)$. $\bar{Z}$ denotes a constant parameter. The logarithm of production is distributed at mean $\ln \bar{Z}$

[11] Deaton and Laroque (1992, 1996) assume a non-zero depreciation rate on the storage.

[12] Deaton and Laroque (1992; 1996) derive the implications for different commodity prices, mostly agricultural crops but also copper, i.e., a non-renewable mineral.

and the standard deviation at σ_z, $\ln Z \sim N\left(\ln \bar{Z}, \sigma_y^2\right)$. The production shock reflects the disturbances from the supply side. It can be recognized as the wars, political events or any unobserved failures of production in the crude oil producing countries. This simplified assumption of exogenous production is standard for short run. For instance, Kilian (2009) also assumed a vertical short-run supply of oil, where supply of oil adjusts infrequently to changes in demand.

As a revised version of the work of Deaton and Laroque (1992; 1996), I also consider that the income cycle follows a stochastic process, such that

$$Y_t = \bar{Y} \exp\left(e_t^y\right), \tag{14}$$

where e_t^y is a normally distributed shock at mean zero and with a standard deviation at σ_y, $e_t^y \sim N\left(0, \sigma_y^2\right)$. $\bar{Y}$ is a constant of the income level. Subsequently, the logarithm of income is distributed at mean $\ln \bar{Y}$ and the standard deviation at σ_y, $\ln Y \sim N\left(\ln \bar{Y}, \sigma_y^2\right)$. A positive income shock can be recognized as a boom in the world economy; on the contrary, a negative income disturbance can be seen as a global recession.

Speculation equilibrium

I solve for the rational expectation model of maximizing expected profits under competitive economy. The arbitrage conditions imply that when storage is positive, the current price equals the expected price in period $t+1$, apart from marginal storage cost. Otherwise, the storage becomes zero when the current price is higher than the marginal gain from storage. That is to say,

$$P_t = \beta E_t\left[P_{t+1}\right] - C, X_t > 0 \tag{15}$$

$$P_t > \beta E_t\left[P_{t+1}\right] - C,\ X_t = 0 \tag{16}$$

where C denotes the cost of storage. β denotes the discount factor.

Thus, the current price depends not only on the current quantity to income ratio as shown in equation (11), but also on the future demand through choosing the storage level X_t according to the future expectation, as in equation (15) and (16).

4.2 The role of income and productions shocks in the rational expectations equilibrium: the mechanism

In order to detect the role of income and production shock on the price in the commodity storage model, in this section, I discuss the results from two simplified storage models: 1) a storage model with a stochastic production process that assumes a constant income $Y_t = \bar{Y}$,[13] 2) a storage model with a stochastic income process that assumes constant production $Z_t = \bar{Z}$.

The rational expectations equilibrium

The solutions from the two commodity storage models specify a rational expectations equilibrium of storage as as function of state variables. In the first storage model with stochastic production, availability A_t is the state variable. Recall that production Z_t is an element of A_t, the optimal storage rule is denoted as $X(A_t)$. In the second model with stochastic income, both availability A_t and income Y_t are state variables. The optimal storage rule is written as $X(A_t, Y_t)$. I solve the two nonlinear rational expectations commodity storage model numerically. Following Miranda and Fackler (2002), I solve the dynamic models by using a spline collocation method for function approximation. The collocation approach in details is summarized in Appendix B.

Simulated moments

To discuss the effect of shocks on equilibrium prices, I simulate prices by employing the optimal storage rule from the two models. The simulation is performed for 20,000 quarters in order to obtain stationary moments. In each period, a production shock or an income shock is drawn from a normal distribution with a mean value of 0 and a standard deviation of 0.1. I impose the same production shock and income shock in absolute values, $\epsilon_t^Y = -\epsilon_t^Z$, for each period.[14] Using simulated shocks and transition functions, I compute state variables in the next period. Subsequently, the storage level can be interpolated by using the optimal storage rule of $X(A_t)$ and $X(A_t, Y_t)$ in each case. After this, I compute the equilibrium price by using the demand function in equation (12).[15] I perform 1,000 repetitions of the simulation in each model.[16] The first 100-period simulations are deleted in order to eliminate the effects of initial values.

Table 3 shows the moments of simulations in the case of production shock and income shock, respectively. The values in Table 3 are the mean value of the simulated series' moments over 1,000 repetitions.

From Table 3, I find that the storage model with a stochastic production process has a stronger and more intensive smoothing effect than the model with a stochastic income process.[17] To be specific, the simulated price in the production-shock model has lower standard

[13]This model is similar to the case assuming a strictly convex price function in Deaton and Laroque (1992).

[14]For comparison, I impose reversed income shocks and production shocks in order to have similar (positive or negative) effects on the price in both models.

[15]Other parameters used for simulation are set as $\gamma = 5$, $\beta = 0.97$ and $C = 0$

[16]The standard error of simulated moments are very robust to changes of larger numbers of repetition in the simulation.

[17]In Table 3, the 95 percent confidence intervals of simulate moments in two models do not overlapped. Thus, the differences are significant.

Table 3: Simulated price moments in the storage model with production shock and income shock

Models	Production (Z) shock	Income (Y) shock
σ_z	0.1	0
σ_y	0	0.1
	(1)	(2)
% of stock outs	3.511	1.186
Moments of P		
mean(P)	0.997	1.039
std(P)	0.226	0.252
1st-order a.c.(P)	0.529	0.513
Moments of A		
mean(A)	1.222	1.192
std(A)	0.176	0.125
1st-order a.c.(A)	0.816	
Moments of X		
mean(X)	0.217	0.192
std(X)	0.144	0.125

Notes: Column (1) shows the moments of price and demand by using the simulated data from the storage model with stochastic production shock. Column (2) shows the moments of simulated data from the storage model with stochastic income shock. The simulations are performed through using estimated parameters for 20,000 periods and 1,000 repetitions. The first 100 periods of simulations are deleted in order to eliminate the effects from the initial values. The measures shown in the table are the average over 1,000 repetitions.

deviation with a value of 0.23 (versus 0.25 in the income-shock model) and a higher persistence with a value of 0.53 (versus 0.51 in the income-shock model), although the production-shock model generates a higher percentage of stock-out with a value of 3.5 percent (versus 1.2 percent in the income-shock model).

Impulse responses

The reason for the stronger smoothing effect in the production-shock model is mainly due to the higher inventory level. As represented in Table 3, I find that the mean level of storage has a value of 0.22 in the production-shock model and 0.19 in the income-shock model.

To support this point, I illustrate the impulse responses of availability, storage, and prices of the two models in Figure 4 using the same initial values in period zero.[18] I impose one standard deviation of positive production shock and negative income shock, respectively, in period one, with $\sigma_Y = \sigma_Z = 0.1$. The shocks are simulated for 20,000 repetitions in period one. The simulation in the subsequent 13 periods assume zero shock on production and income in both cases; thus, $\epsilon_t^Z = \epsilon_t^Y = 0$ for $t > 1$. The series in Figure 4 shows the average over 20,000 repetitions of the simulation across 14 periods.

As represented in the panel (a) of Figure 4, there is an immediate response of availability to the positive production shock in period one in the production-shock model. It simultaneously determines a high storage level in period 1 through the policy function (as shown in the panel (b)). On the other hand, in the income-shock model, the income shock does not have a direct effect on the availability. The availability remains at the same level in period 1.[19] It results in a relatively lower level of storage than in the production-shock case in the shock period in panel (b), although the storage increases due to the negative income shock and thus lower expected price. In the production-shock model with a higher storage level, panel (c) shows a limited drop of price in the shock period and a fast recovery toward a steady state after period 2. Thus, the storage level is more sensitive to the production shock than to the income shock, which introduces a stronger smoothing effect in the production-shock model. This is mainly because income shock is a multiplicative shock on availability, whereas supply shock is an additive shock.

To summarize, the storage model is more sensitive to the production shock than to the income shock. The production shock introduces a more intensive smoothing effect on the equilibrium price, with a low variance and high persistence. In the next section, I will go back to my extended storage model with both production and income shocks, and fit the model to the data. I will also discuss the relative importance of income shock and production shock.

[18] The initial values of state variables for the simulation are $A_0 = 1.1$ and $Y_0 = 1$.

[19] This is consistent with the moments of simulated availability shown in Table 3. In the income-shock model, the availability has a lower mean with a value of 1.19 and with a standard deviation of 0.125, whereas, in the production-shock model, the mean is 1.22 and the standard deviation is 0.18.

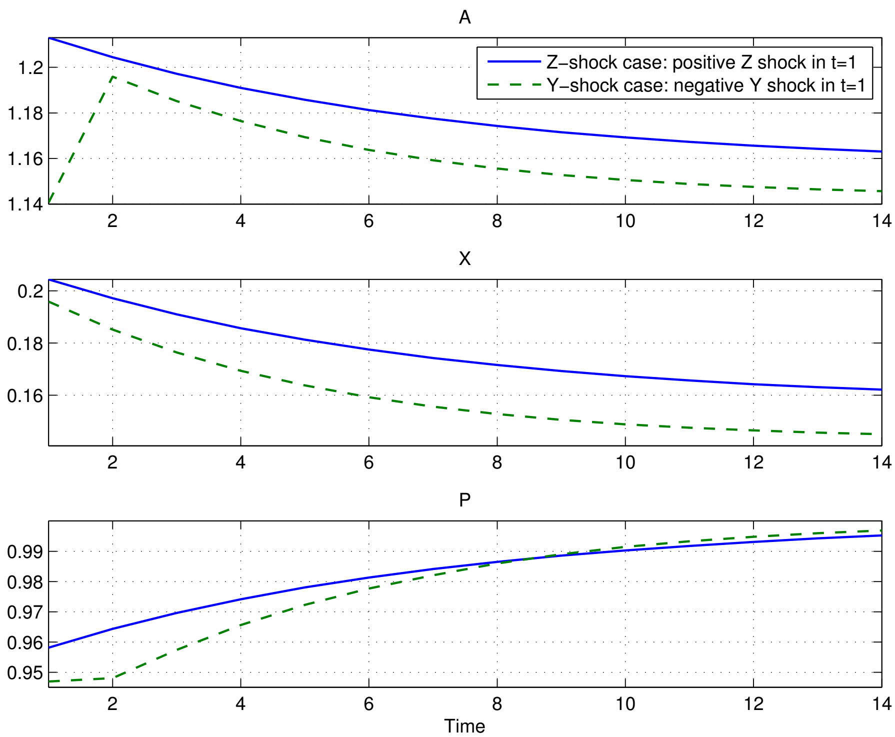

Figure 4: Impulse responses in storage model: The Income-shock and production-shock case

Notes: The initial values of state variables for the simulation are set as $A_0 = 1.1$ and $Y_0 = 1$. One-standard deviation production shock and income shock are imposed in period 1 with $\sigma_Y = \sigma_Z = 0.1$, and let $\epsilon_1^Z = -\epsilon_1^Y$. The shocks are simulated for 20,000 repetitions in period 1, and then the series are simulated for the next 13 periods. The series shown in the figure are the average values over 20,000 repetitions.

5 Matching the extended storage model and the data

In this section, I apply the extended storage model with both a stochastic production and income process to the cyclical components of oil prices, world oil consumption, and global GDP. I employ a moment-matching method for estimation. To discuss the relative fitness to the data, I also compare the estimates and simulated moments from the extended storage model and the model with production shocks. Later, using estimated parameters from the extended storage model, I explain the dynamic behavior of oil prices. Later, I discuss the relative importance of income and production shock through counterfactual analysis. Finally, I compare the impulse response of oil price to the income shock in the extended storage model and the VAR analysis.

5.1 Estimation

Calibrated parameters

By assuming an exogenous process on income and production in equation (13) and (14), some parameters can be calibrated independently from the storage model. Through the construction of BN decomposition, the logarithm values of oil cycles fluctuate around 0. It is reasonable then to have $\bar{Y} = 1$ and $\bar{Z} = 1$. Moreover, knowing the log-normal distribution of income with $\ln Y \sim N\left(\ln \bar{Y}, \sigma_Y^2\right)$, I calibrate the standard deviation of income shock at $\sigma_Y = 0.02$ through using an observed GDP cycle.[20]

SMM

Subsequently, I employ a SMM estimation method introduced by Lee and Ingram (1991), which estimates structural coefficients – the demand elasticity parameter γ, production volatility σ_z, and discount rate β. The SMM estimation is implemented by first solving the extended storage model (with a stochastic production and income process) through using a collocation method for a set of parameters, and then simulating series of price $\tilde{P}_t$, oil demand $\tilde{Q}_t$, and income $\tilde{Y}_t$ by using the policy function of storage.[21] The optimal coefficients are determined when they minimize the weighted sum squares of the difference between empirical and simulated data moments, such that the following equation can be formulated:

$$\hat{\theta} = \arg\min_{\theta} M\left(\theta\right)' W M\left(\theta\right). \tag{17}$$

$\theta \equiv [\gamma, \beta, \sigma_Z]$ is the structural coefficient vector of interest, which is a $l \times 1$ ($l = 3$) vector. $M\left(\theta\right)$ is a $k \times 1$ moment condition which is the difference between empirical and simulated

[20]Since the production of oil is unobserved, the standard deviation of production shock σ_z can not be calibrated and is estimated using the moment-matching method.

[21]I assign $\bar{Z} = 1$ and calibrated parameters of $\bar{Y} = 1$ and $\sigma_Y = 0.02$ using world GDP data.

data moments, such that

$$M(\theta) = \frac{1}{T}\sum_{t=1}^{T} m(P_t, Q_t, Y_t) - \frac{1}{N}\sum_{t=1}^{N} m\left(\tilde{P}_t(\theta), \tilde{Q}_t(\theta), \tilde{Y}_t(\theta)\right), \tag{18}$$

where T denotes the number of observation; N denotes the sample size of simulated data, and $N = T \times H$ where $H \geq 1$.[22] I employ $H = 50$ in the estimation.[23] $m(P_t, Q_t, Y_t)$ and $m\left(\tilde{P}_t(\theta), \tilde{Q}_t(\theta), \tilde{Y}_t(\theta)\right)$ are moments computed by using observed data and simulated data.[24] In equation (17), W denotes the optimal weighting matrix evaluated as the inverse of the covariance matrix of empirical data moment $m(P_t, Q_t, Y_t)$; accordingly, the following equation applies:

$$W = \frac{1}{T}\left[m(P_t, Q_t, Y_t)' \, m(P_t, Q_t, Y_t)\right]^{-1}. \tag{19}$$

In the SMM estimation, the moment function includes the mean, variance, auto-covariance of price and quantity demand, and income-quantity covariance, such that the following equation applies:

$$m(\theta) = \begin{bmatrix} \left(P_t - \bar{P}\right)^2, \left(Q_t - \bar{Q}\right)^2, \left(Y_t - \bar{Y}\right)\left(Q_t - \bar{Q}\right), \\ \left(P_t - \bar{P}\right)\left(P_{t-1} - \bar{P}\right) \\ \left(Q_t - \bar{Q}\right)\left(Q_{t-1} - \bar{Q}\right) \end{bmatrix}. \tag{20}$$

Furthermore, the asymptotic distribution of θ is given by

$$\sqrt{T}\left(\hat{\theta} - \theta_0\right) \rightarrow^d N(0, \Omega),$$

where Ω denotes the $k \times k$ covariance matrix, such that

$$\Omega \equiv \left(1 + \frac{1}{H}\right)\left[\frac{\partial M(\theta)}{\partial \theta} W \frac{\partial M(\theta)}{\partial \theta'}\right]^{-1}.$$

Estimation results

Following equation (17)–(20), I obtain the SMM estimates as shown in Table 4. The first column in Table 4 shows the estimated coefficient, and the second column presents the standard error of the estimates.

The coefficient γ, which measures the relative changes of price with respect to the change

[22] Following Michaelides and Ng (2000), the data are simulated for $T \times H \times 1.1$ periods, and the first 10 percent-period simulated data is trimmed.

[23] Michaelides and Ng (2000) found a good sample performance when the simulated sample is approximately 10 times as large as the actual data. I also find that the estimates are robust to different numbers of H, when $H \geq 10$.

[24] In the SMM estimation, I assign the same random values for the production shock and income shock in each iteration. This is in order to satisfy the property of "stochastic equicontinuity" for simulation estimators as shown in McFadden and Ruud (1994).

Table 4: SMM estimates of parameters for the extended storage model

	Extended storage model		Z-shock storage model	
	coeff.	s.e.	coeff.	s.e.
γ	5.215	(1.343)	3.156	(0.197)
β	0.989	(0.006)	0.975	(0.196)
σ_Z	0.055	(0.016)	0.006	(0.021)
J-statistics	1.064		12.605	
p-value	58.8%		0.6%	
Degree of freedom	2		3	

Notes: The heteroskedasticity and autocorrelation consistent (HAC) standard errors are shown in parentheses.

in the Q to Y ratio, is estimated at 5.22, which is statistically significant with a 95 percent confidence interval. This implies a price elasticity of oil demand of $-\frac{1}{\gamma} = -\frac{1}{5.22} = -0.19$.[25] This estimate is close to the results of other scholars who have studied the oil price elasticity of demand. Dahl (1993), Gately and Huntington (2002), Cooper (2003), and Dées et al. (2007), among others, estimated single-equation models of oil demand, and obtained demand elasticities for crude oil between -0.2 and -0.6. Alhajji and Huettner (2000) estimated a structural model of crude oil market and arrived at a price elasticity of demand of -0.25.

The quarterly discount factor is estimated to be 0.99 (with a standard error of 0.01). It implies a quarterly interest rate at 0.01, and a annual interest rate at 0.04.[26] This estimated interest rate is close to the results in Deaton and Laroque(1992; 1996), in which the annual interest rate, for example, is estimated to be 0.05 for copper. Furthermore, I obtain a statistically significant production volatility of 0.06, which is five times larger than the volatility of income shock.

In addition, an overidentifying test is implemented to test the model specification. The Sargan-Hansen's J-statistic is computed as the fraction of the optimal value of the SMM objective function. The J-statistic follows a Chi-square distribution at $k - l$ degrees of freedom, as shown in the following equation:

$$T\left(1+\frac{1}{H}\right)\left[M\left(\hat{\theta}\right)' WM\left(\hat{\theta}\right)\right] \xrightarrow{d} \chi^2\left(k-l\right).$$

In the case of this analysis, I have $k = 5$ and $l = 3$, and thus $k - l = 2$. Table 4 shows a J-statistic of 1.064. This suggests that we can not reject the null hypothesis that model

[25]The standard error for the price elasticity of demand is 0.014, which is computed through the delta method.

[26]The discount factor is function of interest rate r, such that $\beta = \frac{1}{1+r}$.

moments match data moments, $H_0 : E[M(\theta)] = 0$, at a significance level of 5 percent. Thus I conclude that the model is correctly specified and the parameters are estimated consistently.

As a comparison, I also estimate the storage model with only production shock as in Deaton and Laroque (1992) using SMM.[27] The estimates are shown in the last column in Table 4. I find that γ is estimated significantly at 3.16 with standard error 0.20. I employ a Z-test to investigate the equality of estimate γ from the extended storage model and the production-shock model.[28] Using the estimates of γ from these two models, I compute a Z-statistic at 1.52 which implies a p-value at 12.9 percent. Thus I can not reject the null hypothesis of equal γ estimated from the two models at 10 percent significance level. Next, the estimated discount rate β in the production-shock model is 0.98, which is very close to the result from the extended storage model. However, in the production-shock model, the production shock volatility is estimated statistically insignificant (0.006 with standard error 0.02). Meanwhile, the Sargan-Hansen's J test is computed 12.61 with p-value at 0.6 percent, which suggests a rejection of the null hypothesis of model moments matching data moments at 5 percent significance level. The last two results reveal that the commodity storage model with only production shock may be misspecified. The empirical results also show that the extended commodity storage model represents a major improvement over the original Deaton and Laroque (1992)'s model for the crude oil market.

5.2 Fitness of the storage model

Using the estimated parameters from the extended storage model in Table 4, I discuss in this section the applicability of the extended storage model to the data by comparing the moments of empirical data and the simulated data from the model. I also compare the moment of simulated data from the extended storage model and the storage model with production shock only to discuss which model has better fitness to the data. The results are presented in Table 5.

The simulations are performed by using estimated parameters for 20,000 periods and 1,000 repetitions. The first 100 periods of simulations are deleted in order to eliminate the effects from initial values. Column 1 in Table 5 shows the data moments of oil price cycles and oil consumption cycles. Column 2 shows the moments of simulated series from the extended storage model that allow for positive storage. Column 3 shows the moments of simulated series that impose zero storage in all periods. Column 2 and 3 show the mean value of

[27] The moment function includes the mean, variance, and auto-covariance of price and quantity demand.

[28] The Z-statistic is compute as $Z = \frac{\gamma_1 - \gamma_2}{\sqrt{(SE\gamma_1)^2 + (SE\gamma_2)^2}}$, where γ_1 and γ_2 denote the estimates from the extended storage model and from the production-shock model respectively, and $SE\gamma$ denotes the standard error of γ.

Table 5: Moments of oil price and simulated prices in the extended storage model and the storage model with production shock only

	Data	Extended storage model		Z-shock storage model
		$X \geq 0$	$X = 0$	$X \geq 0$
	(1)	(2)	(3)	(4)
% of stock outs		3.191	100	0.229
mean(P)	1.005	1.000	1.043	1.000
std(P)	0.113	0.124	0.308	0.011
1st-order a.c.	0.591	0.536	-0.001	0.514
skewness	3.547	3.422	0.911	0.364
kurtosis	21.191	23.046	4.506	3.630
mean(Q)	1.001	1.002	1.002	1.000
std(Q)	0.024	0.021	0.055	0.003
1st-order a.c.	0.402	0.515	-0.001	0.515
skewness	0.097	-1.511	0.165	-0.311
kurtosis	3.437	7.969	3.049	3.557
corr(Q,Y)	0.301	0.367	3.049	–

Notes: Column 1 presents data moments by using cyclical components of the oil price and the world oil consumption. Column 2 shows the moments of price and demand by using the simulated data from the extended storage model. Column 3 shows the moments of simulated data assuming storage is zero in all periods. Column 4 shows the moments of price and demand by using the simulated data from the storage model with production shock only. The simulations are performed using estimated parameters for 20,000 periods and 1,000 repetitions. The first 100 periods of simulations are deleted in order to eliminate the effects from initial values.

moments over 1,000 repetitions.

In Table 5, column 1 and 2 reveal a good fit between the model and the data. The columns show that the storage model generates a price with a mean value of 1.00 and with a standard deviation of 0.12, which is close to the data moments with a mean value of 1.01 and a standard deviation of 0.11. Furthermore, the extended storage model is able to capture the persistence in the oil price. The persistence is 0.54 for the simulated price and 0.59 for the price cycle data.

Although the skewness and kurtosis are not included in the moments condition in equation (20) for the SMM estimation, the extended storage model reveals a good capability to reproduce the higher moments as in the data. As Table 5 shows, the skewness is 3.42 for the simulated price and 3.55 for the data. Similarly, the kurtosis is 23.05 for the simulated price, and 21.19 for the data. The close match to the data skewness and kurtosis also confirms the good fit of the model to the data.

Column 4 in Table 5 also shows the simulated moments of the storage model with single production shock using estimates from Table 4. Comparing to column 2, I find that the production-shock model is not able to match many moments in the data, especially the standard deviation, skewness and kurtosis of the oil price and consumption. This also reveals that the production-shock model does not fit well to the data, whereas the extended storage model with both production and income shocks has better fit.

Furthermore, a comparison between column 2 and 3 in Table 5 reveals the effect of speculative behavior, wherein column 3 presents the moments of simulated prices that impose zero storage for all simulation periods. I use the same parameters for simulation in column 2 and 3.

The extended storage model behaves similar to that of Deaton and Laroque (1996), such that the storage dampens shocks by means of lowering the price volatility and introducing persistence. As Table 5 illustrates, the volatility of simulated prices is 0.12 with possible storage and 0.31 without storage. The persistence of the simulated price is 0.54 with possible storage and close to zero when storage is impossible.

These results reveal that the storage has a smoothing effect that mitigates shocks. Intuitively, when the current price is high, caused by a high (low) GDP (production), it is most likely that the future GDP (production) shock is low (high) due to the zero shock persistence. Since the income (and production) process is mean reverting, speculators expect a lower future price, and thus have incentives to reduce storage in the current period. This leads to a decrease of oil availability in the market in the subsequent period. It then implies a high future price following the high price in the current period. Therefore, speculative behavior smoothens the price cycles.

In Figure 5, I also show a simulated price of oil from the storage model as an instructive illustration, (see Deaton and Laroque (1992)). The series is simulated by using estimated parameters. The simulation is implemented for 300 periods. The first 100-period simulations are deleted in order to eliminate the effects from initial values.[29] Figure 5 shows marked resemblances in the features of the simulated price to the price cycle data that are shown in Figure 1. From Figure 5, one can observe that the model generates occasionally large upward spikes (see plot in Figure 1). Moreover, the model is able to produce the low-variance phase more often when the oil price cycle is low.

5.3 The importance of income and production shocks

Using estimated parameters of the extended storage model, I perform a counterfactual analysis to discuss the relative importance of the income and production shocks as a means of

[29]While simulating the price of oil, I use world GDP cycle data as the income process.

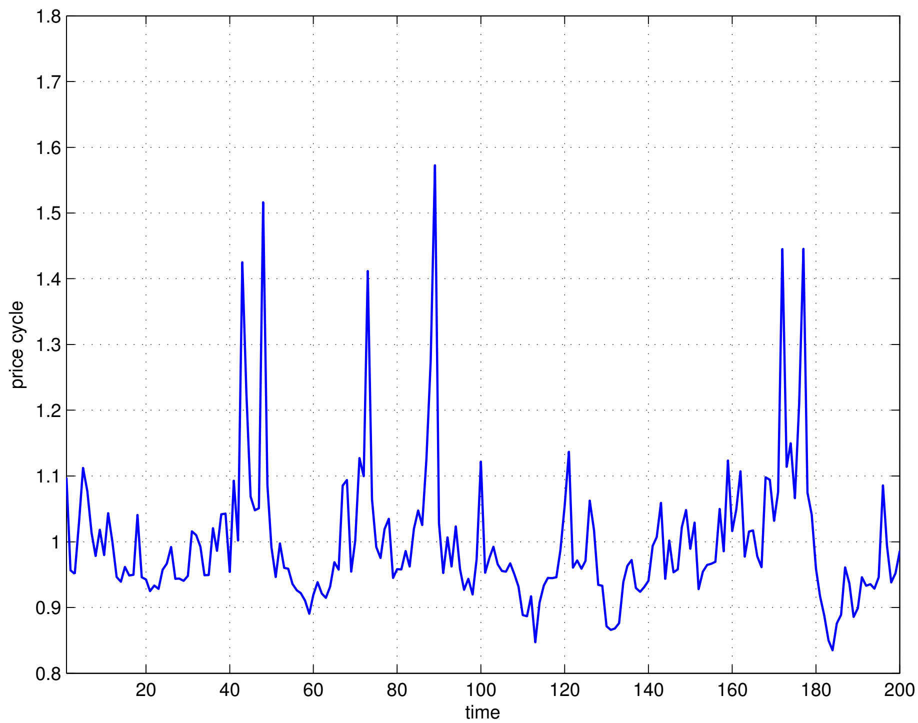

Figure 5: Simulated price of oil from the extended storage model

Notes: The price of oil cycle is simulated for 300 periods. The first 100-period simulations are deleted in order to eliminate the effects from initial values. In the simulation, the global GDP data is used as the exogenous process of income.

Table 6: Moments of simulated prices with zero production and income shock

	$\epsilon_z, \epsilon_y \neq 0$ (1)	$\epsilon_z = 0, \epsilon_y \neq 0$ (2)	$\epsilon_z \neq 0, \epsilon_y = 0$ (3)
mean(P)	1.000	1.000	0.999
std(P)	0.124	0.055	0.118
var(P)	0.015	0.003	0.013
corr(P_t,P_{t-1})	0.536	0.495	0.539
cov(P_t,P_{t-1})	0.008	0.001	0.007

Notes: This table shows the moments of simulated prices in different scenarios. Column 1 presents the moments of simulated price in the extended storage model with both income and production shocks. Column 2 refers to the case that assumes zero production shock in the simulation. Column 3 refers to the case that assumes zero income shock in the simulation. For the simulation, in column 2 and 3 I use the same policy function of storage as in column 1. The simulations are performed through using estimated parameters in Table 4 for 20,000 periods and 1,000 repetitions. The first 100 periods of simulations are deleted in order to eliminate the effects of initial values.

explaining the variance and autocovariance of the oil price cycle for the period 1986–2009. I also employ the impulse responses so as to illustrate the response of the oil price cycle to income shock, and so as to compare it with the evidence from the data in the VAR analysis in Section 3.

Counterfactual analysis

Table 6 presents a counterfactual analysis in three different cases of price simulation. The first column presents the moments of simulated price cycles in the benchmark model with a stochastic production and income process. Estimated parameters in Table 4 are used for simulation. The values are consistent with those in column 2 in Table 5.

I derive two counterfactual exercises. First, I impose zero production shock over the simulation period ($\epsilon_Z = 0$), and I use identical estimated parameters, as in the benchmark model. Moreover, I employ the policy function of storage derived from the benchmark model for interpolation and simulation. The moments of the simulated price with zero production shock is shown in column 2 in Table 6. Second, I let the income remain constant at its mean value over time (thus $\epsilon_Y = 0$), and similarly I use the same parameters and the policy function as in the benchmark model. The moments of the simulated price with zero income shock are shown in column 3 in Table 6.

As column 2 illustrates, setting a zero production shock strongly impacts the volatility and persistence of the oil price cycle. With a similar mean of 1.00, the variance of the simulated price is 0.003 with zero production shocks–much lower than the 0.015 in the benchmark case. These results indicate that the production shocks contribute 80 percent of the price variance in the benchmark case. At the same time, the autocovariance of the simulated price with a zero production shock is 0.001, versus 0.008 in the benchmark case. Thus the production shock accounts for 87.5 percent of the autocovariance of price in the benchmark case.

However, the income shock moderately impacts the volatility and persistence of the price cycles. The variance and autocovariance of the simulated price with zero GDP shock are 0.013 and 0.007, which, respectively, account for 13 percent and 12 percent of the corresponding moments in the benchmark case.[30]

The higher importance of production shock to the oil price cycle is mainly because of the large estimated production volatility (see Table 5). Due to the unobservable crude oil production in the data, the SMM estimation method searches for the proper production shock volatility (and other system parameters) in order to match model-implied moments to the data moments. On account of the GDP volatility of 0.01 through the calibration, the estimated production volatility is obtained when the simulated price have the closest persistence (together with other moments) to that of the data (0.59). As I explained in Section 4.2, the production shock motivates storage and increases the smoothing effect. Subsequently, the SMM estimates a production-shock volatility of 0.06—five times as large as the volatility of GDP shock; in other words, the large estimated volatility of production shock reveals the importance of the production shock to the oil price cycles.

5.4 Comparison with the VAR model

In order to observe whether the extended storage model is in line with the empirical evidence in the VAR analysis, I compute impulse responses of the oil price cycle to income shocks (in the extended storage model), and I compare them with the data of oil cycles from the VAR model in Section 3. Figure 6 depicts the impulse responses of income shocks on the cyclical components of world GDP and oil price in the storage model and VAR model. The impulse responses in the VAR model are identical to those in Figure 3. I impose the same positive income shock in period 0 in both the storage model and the VAR model, and I assume the shock will return a value of 0 in the subsequent 14 periods. The production shock is set to be 0 for all the periods. Panel (a) illustrates identical response of initial world GDP at 0.67 percent in period 0, and panel (b) plots the impulse responses of price.

In the storage model, as in equation (14), the income process is assumed to have 0 persistence. Thus, the impulse response on world GDP, as shown in panel (a), returns to 0 percent right after the period with positive income shock. Due to the autoregressive feature of the VAR model, the impulse response of income shock on GDP process gradually fades

[30] The sum of the contribution to the overall price variance and autocovariance in the two counterfactual cases does not equal 100 percent. This is mainly because the simulations of price vary in each case of the 1,000 repetitions, and I compute the fraction using a mean value of the 1,000 repetitions of simulations.

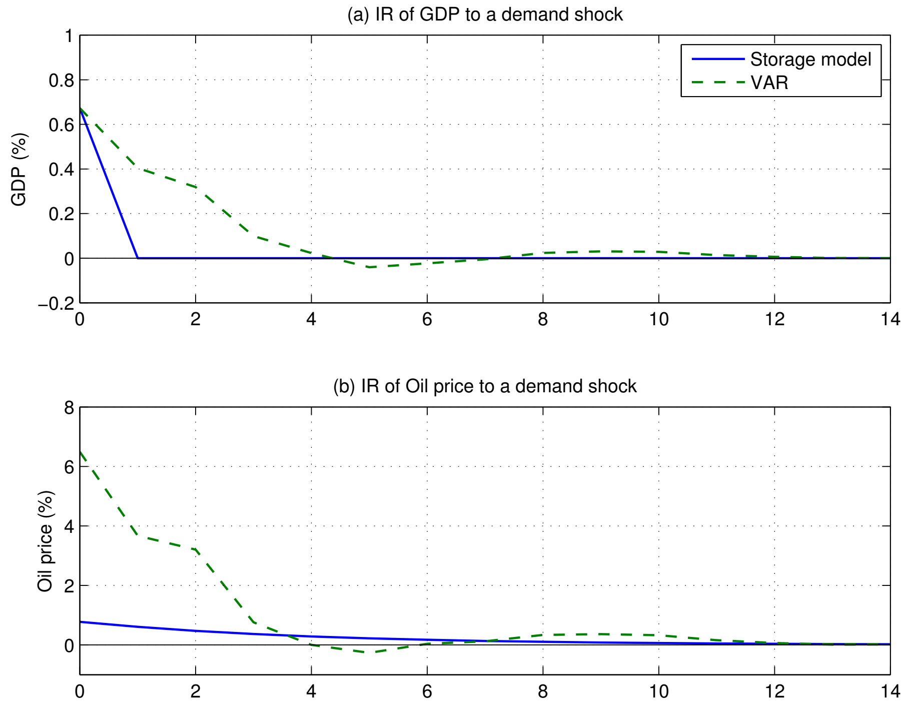

Figure 6: Impulse responses of GDP and oil price cycle with respect to income shock in the storage model and VAR model

Notes: Panel (a) plots the impulse responses of GDP with respect to the same positive income shocks in period 0 in the storage model and VAR model. Panel (b) shows the impulse responses of price cycles with respect to positive demand shocks (as shown in the panel (a)) in the storage model and VAR model. The income shocks are simulated for 5,000 repetitions in period 0. The shocks returns to 0 in the subsequent 14 periods. The production shock is assumed to be 0 for all the periods. Figures plot the median value for the 5,000 repetitions of the simulations.

out after 12 periods.

By imposing the same income shocks in period 0 in both the storage model and VAR model, I find that the oil price cycle has limited impulse response to world GDP shock in the storage model. Panel (b) shows that facing the same positive demand shocks and zero supply shock, the oil price cycle increase only 1 percent in the storage model, compared to 6.30 percent in the VAR model.

In summary, the revised commodity storage model, extended with both stochastic production and income processes, is capable of capturing the large volatility and persistence in the cyclical component of the oil price. The autocorrelation of the oil price cycle can be fully attributed to the speculation effect. However, the model generates a moderate response towards income shocks.

6 Conclusions

In this paper I employ a multivariate method to extract the cyclical component of the oil price. I find a large and positive effect of global GDP shock on the oil price cycles in a VAR model. I apply an extended commodity storage model with both income shock and production shock to the cyclical component of oil prices, after all long-term trends have been removed. This model is estimated using SMM for the period 1986–2009.

I obtain encouraging results in several respects. First, I find significant and meaningful coefficients that are estimated from the commodity storage model. Second, through employing estimated parameters, the model is capable of replicating some stylized features of oil cycles—in particular, the volatility and persistence of the oil price cycle. Comparing to the model with only production shock, the extended storage model has better fitness to the data. Furthermore, in the extended storage model, income shocks explain 13 percent of the variance in oil prices, whereas production shocks explain 87 percent. This is due to the large volatility of production and the intensive smoothing effect on the equilibrium price that is triggered by the production shock. A comparison of impulse responses between the extended storage model and the VAR model shows that the GDP shock generates more moderate effect of the oil price cycles in the extended storage model than in the VAR analysis.

The limitations of this paper are mainly related to the simplified assumption of a stationary stochastic income process. The world GDP as a measure of world income is concluded as a non-stationary series. For further extension of my model, it would be advisable to assume an income process with both transitory and permanent components in an empirical storage model. I can then assess the income effect on the price of crude oil and compare with the results in the current paper. Through such an estimation, I will be able to evaluate the fraction of transitory and permanent components in the income process.

Furthermore, this paper does not have a detailed setup for a speculator, such as the OPEC core, with market power on controlling the oil price through production. Therefore, possible research can be extended to an endogenous production process in the commodity storage model. From the estimation of such a model, I may be able to show the importance of having the endogenised production process. The estimates will also give an indication of the market power of the speculator.

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Appendix

A The Implementation of the Beverage-Nelson decomposition: a state-space approach

This appendix summarizes the state-space approach for the BN decomposition with detected number of cointegrating relationship as discussed in Cochrane (1994) and Morley (2002).

I decompose the cyclical components using estimates from the VEC model as equation (6) with one cointegration relationship. Due to the existence of one cointegration relationship among variables, it is possible to identify speed of adjustment coefficients, denoted as γ, and cointegration vector, denoted as α, using maximum likelihood, such that

$$\hat{\Pi} = \hat{\gamma}\hat{\alpha},$$

where $\hat{\Pi}$ denotes the estimates of Π in the VEC model in equation (6). $\hat{\gamma}$ is a 3×1 coefficient matrix. $\hat{\alpha}$ is a 1×3 structure coefficient matrix for the long-run stationary relationship

$$\hat{\alpha}\mathbf{X}_t = \hat{\alpha}_y \ln Y_t + \hat{\alpha}_q \ln Q_t + \hat{\alpha}_p \ln P_t \sim I(0),$$

where $\hat{\alpha} = \begin{bmatrix} \hat{\alpha}_y & \hat{\alpha}_q & \hat{\alpha}_p \end{bmatrix}$.

Cochrane (1994) suggests a stylized method by transforming the VEC model into an AR(1) format when computing the trend and cyclical components. Following Cochrane (1994) I transform the VEC in (6) into an AR(1) format such that

$$\begin{bmatrix} \triangle\mathbf{X}_t \\ \hat{\alpha}\mathbf{X}_t \end{bmatrix} - \hat{\mu} = \hat{\mathbf{B}}\left(\begin{bmatrix} \triangle\mathbf{X}_{t-1} \\ \hat{\alpha}\mathbf{X}_{t-1} \end{bmatrix} - \hat{\mu}\right) + \begin{bmatrix} u_t \\ \hat{\alpha}u_t \end{bmatrix}, \tag{21}$$

where

$$\hat{\mu} = \left(\mathbf{I} - \hat{\mathbf{B}}\right)^{-1} \begin{bmatrix} \hat{\beta} \\ \hat{\alpha}\hat{\beta} \end{bmatrix}$$

$$\hat{\mathbf{B}} = \begin{bmatrix} \hat{A}_1 & \hat{\gamma} \\ \hat{\alpha}\hat{A}_1 & \hat{\Pi} + 1 \end{bmatrix}.$$

$\hat{\alpha}$, $\hat{\beta}$, $\hat{\gamma}$, $\hat{A}_1$ and $\hat{\Pi}$ are the estimates of coefficient in the VEC model (6).

Beveridge and Nelson (1981)define the trend component of $\mathbf{X}_t$ as the expectation of the h-step ahead forecast where $h \to \infty$. Following Cochrane (1994), the trend component is computed as

$$\mathbf{X}_t^T = \mathbf{X}_t + \left[\begin{array}{cc} \mathbf{I}_{3\times3} & \mathbf{0}_{3\times1} \end{array} \right] \hat{\mathbf{B}} \left(\mathbf{I} - \hat{\mathbf{B}}\right)^{-1} \left(\left[\begin{array}{c} \triangle \mathbf{X}_t \\ \hat{\alpha} \mathbf{X}_t \end{array} \right] - \hat{\mu} \right),$$

and the cyclical component is computed as

$$\begin{aligned} \mathbf{X}_t^C &= \mathbf{X}_t - \mathbf{X}_t^T \\ &= -\left[\begin{array}{cc} \mathbf{I}_{3\times3} & \mathbf{0}_{3\times1} \end{array} \right] \hat{\mathbf{B}} \left(\mathbf{I} - \hat{\mathbf{B}}\right)^{-1} \left(\left[\begin{array}{c} \triangle \mathbf{X}_t \\ \hat{\alpha} \mathbf{X}_t \end{array} \right] - \hat{\mu} \right). \end{aligned}$$

B Solving the Nonlinear Rational Expectations Commodity Market Model

This appendix summaries the collocation method of solving a simplified rational expectations commodity market model with one state variable and one control variable following Miranda and Fackler (2002). The extended model in this paper is solved in the same logic.

The simplified model

Let us consider a simple commodity storage model with a stochastic production process, where at beginning of period t, the availability of the commodity is A_t. Meanwhile, suppose that an amount Q_t is sold to consumers at a market clearing price $P_t = P\left(\frac{Q_t}{\bar{Y}}\right)$. At each time the producer can either produce or store the product, therefore I have that in each period the availability equals the sum of storage and consumption, $A_t = X_t + Q_t$. Speculators observe the availability and make decisions on the storage amount X_t following an arbitrage equilibrium condition, as shown in equation (15) and (16), derived from maximization of expected profit. Then I can write the complementarity problem as follow:

$$\begin{aligned} f_t &= \beta E_t \left[P\left(\frac{A_{t+1} - X_{t+1}}{\bar{Y}}\right)\right] - P\left(\frac{A_t - X_t}{\bar{Y}}\right) - C \qquad (22) \\ X_t &\geq 0, f_t \leq 0, \\ X_t &> 0 \Longrightarrow f_t = 0 \\ X_t &= 0 \Longrightarrow f_t < 0. \end{aligned}$$

Finally, the storage in the next period X_{t+1} depends on the current states A_t and Y_t, the control variable X_t and the exogenous production shocks e_{t+1} which is realized after time t.

Then the transition function of the state variable can be written as follow

$$A_{t+1} = g\left(A_t, X_t, e_{t+1}\right) = X_t + \bar{Z}\exp\left(e_{t+1}\right)$$

In this problem, the state space is $A \subseteq R^{d_a}$, and the response space is $X \subseteq R^{d_x}$. The production shock e is normally distributed with mean 0 and variance σ^2.

Collocation method

To solve this rational expectation model with non-smooth policy function, I first specify the state variable with N number of nodes, such that A_i for $i = 1, 2, ..., N$. After that, I approximate the equilibrium price function $P\left(\right)$ in (22) as follow

$$P\left(A, X\left(A\right)\right) = \sum_{j=1}^{N} c_j \phi_j\left(A\right), \tag{23}$$

where the equilibrium price function is a linear combination of known basis function ϕ_j with coefficients c_j for $j = 1, 2, ..., N$.[31]

Then I can rewrite the original complementary problem into to the form

$$\begin{aligned} f\left(A_i\right) &= \beta \sum_{k=1}^{K} \sum_{j=1}^{N} w_k \left[c_j \phi_j\left(X\left(A_i\right) + \bar{Z}\exp\left(e'\right)\right)\right] - P\left(A_i - X\left(A_i\right)\right) - C \qquad (24) \\ X\left(A_i\right) &\geq 0, f\left(A_i\right) \leq 0, \\ X\left(A_i\right) &> 0 \Longrightarrow f\left(A_i\right) = 0 \end{aligned}$$

for each $i = 1, 2, ..., N$. In equation (24), the random production shock e in the transition function is substituted with discrete approximation of e_k and probabilities w_k for $k = 1, 2, ..., K$. This method transfers the model into N nonlinear equations and N unknown coefficients c_j for $j = 1, 2, ..., N$.

I use a two-layer-iteration-loop method to solve equation (24). First, I give an initial guess of coefficient c_j for $j = 1, ...N$. In the inner loop, with given initial guess of coefficients, I find the optimal solution of the control variable X at each state nodes A_i in equation (24).[32] Second, using optimal solution of the control variable from the inner loop, I am able to compute the updated coefficient c_j using equation (23) in the outer loop. After that, the newly updated coefficient enters the inner loop to compute optimal control variable at each state variable again. The iteration carries on until the coefficients convergence.

[31] I employ cubic spline method for the basis function.

[32] Following Miranda and Fackler (2002), I solve the complementarity problem using min-max root-finding method.